Topics on experimental and policy design with spillover effects

Davide Viviano
Harvard University

November 17, 2026

Introduction

- Spillovers are everywhere: E.g., more than 50% of papers in NBER SI dev programs in past four years measure/account for spillovers. Important when thinking of scaling up intervention.
- Examples
  - Information campaigns on online platforms [Bond et al., 2012]
  - Informing farmers in rural villages about insurace [Cai et al., 2015]
- In this presentation
  - An overview of the literature and some challenges
  - Design experiments to capture spillovers
  - Design of policy decisions with networks

Some of the papers I will cover today

Policy design with unknown interference
with Jess Rudder
Design experiments to improve policies when network links are unknown.

Causal clustering
with Lihua Lei, Guido Imbens, Brian Karrer, Okke Schrijvers and Liang Shi
Choose clusters to balance spillover bias and sampling variance.

Policy targeting under network interference
Learn whom to treat to maximize welfare in the presence of spillovers.

Content

1. Setup and overview of recent literature
2. Topics on network experiments
3. Policy design

Potential outcomes with spillovers

- $\left(Y_i, D_i, X_i\right)$: outcome, (binary) treatment and covariate of unit i
- SUTVA assume independence, so that $\left(Y_i(1), Y_i(0)\right)$ are potential outcomes that only depend on **ego** treatment
- With spillovers, one formalization of potential outcomes is to write

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Total effect

$$\frac{1}{N}\sum_{i=1}^{N}\mathbb{E}\Big[Y_i(\mathbf{1}) - Y_i(\mathbf{0})\Big]$$

Compare treating everyone with treating no one.

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Direct effect

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Change a unit's own treatment while holding everyone else's fixed.

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Spillover effect

$$\frac{1}{N}\sum_{i=1}^{N}\mathbb{E}\Big[Y_i(d, \mathbf{d}_{-i}) - Y_i(d, \mathbf{d}'_{-i})\Big]$$

Change others' treatments while holding the unit's own treatment fixed.

All sorts of complications:

- **Many potential outcomes:** With N units and binary treatment, there are 2^N possible assignments.
- **Multiple equilibria:** Counterfactuals may depend on which equilibrium is selected (Kline and Tamer, 2020).
- **Missing network data:** Recording every link can be costly. Aggregated reports offer an alternative (Breza et al., 2020).
- **Endogenous networks:** Unobserved traits can affect both link formation and outcomes (Goldsmith-Pinkham and Imbens, 2013).

We will focus on a few of these challenges.

Some approaches (with a single network)

- **Design-based identification**: use the randomization of treatments. (Hudgens and Halloran, 2008; Aronow and Samii, 2017; Borusyak and Hull, 2023; Athey et al., 2018; Basse et al., 2019; Menzel, 2025),
  - **Inverse probability weighting**
  - **Recentering**
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  (e.g. Munro et al., 2025; Brock and Durlauf, 2001; Bhattacharya et al., 2024; De Paula et al., 2018; La Ferrara et al., 2026)
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- **Experimental design: clustering and saturation experiments**

Exposure mapping: definitions

- Exposure mapping defines a function that maps from alter-ego treatments to some real value (Aronow and Samii, 2017)

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  Ex1 Outcomes depend on the share of treated friends

$$g_i(\mathbf{d}_{-i}) = \frac{1}{\#\text{friends of i}} \sum_{k \text{ is friend with i}} D_k$$

  Ex2 Outcomes depend on effect within a given radius.

  Ex3 Spillovers only occur within clusters

- Idea is to "truncate" the dependence when there is complete dependence in a single network (Leung, 2022a).

Allowing for complete (but decaying) dependence

- A related but different approach is to consider explicit models of outcomes as a function of the **outcomes** of other units. For instance, a common model (Manski, 1993; Bramoullé et al., 2009)

$$\mathbf{y} = \alpha\mathbf{1} + \rho\mathbf{A}\mathbf{y} + \mathbf{X}\gamma + \mathbf{A}\mathbf{D}\delta + \boldsymbol{\varepsilon}, \qquad E[\boldsymbol{\varepsilon} \mid \mathbf{D}, \mathbf{A}] = 0.$$

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- Main difficulty: $\mathbf{A}\mathbf{y}$ is determined at the same time as $\mathbf{y}$.
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- Main difficulty: **Ay** is determined at the same time as **y**.
- Aggregate/group-level shocks challenge identification: **Ay** may be perfectly correlated with an aggregate group-level component (Manski, 1993).
- Common identification strategy is to use friends of friends as an instrument. Identification requires that some friends of friends are not direct friends + model must be correct (Bramoullé et al., 2009)

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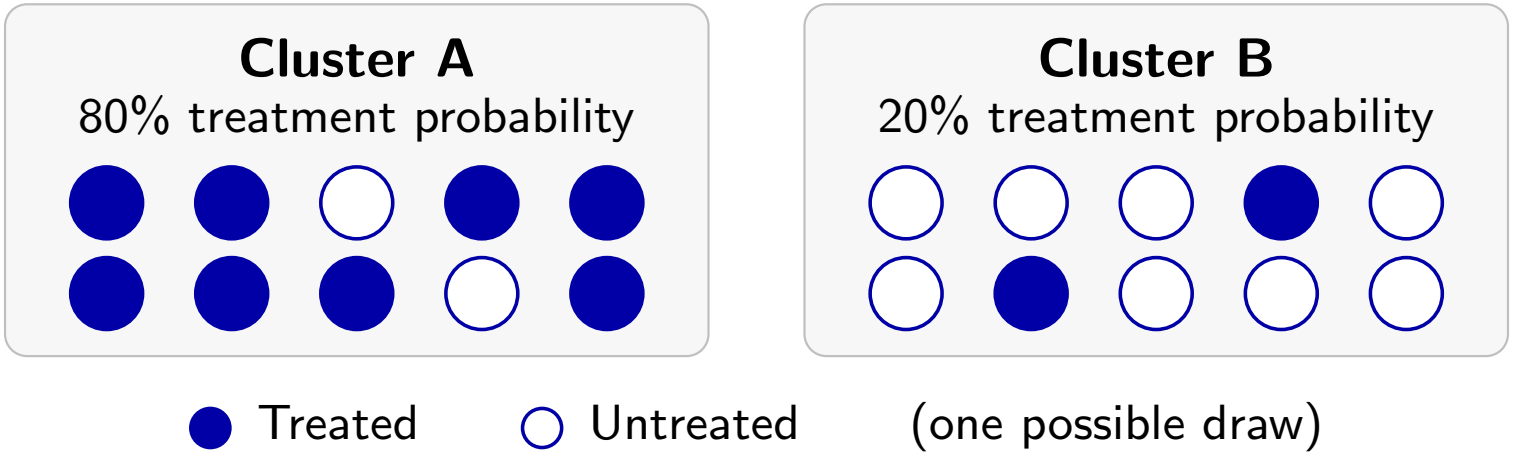
- Partition the population in K many clusters (regions, villages, etc)
- Define $\pi_1, \cdots, \pi_K$ cluster level treatment probabilities
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- Cluster experiments: assume that $\pi_k \in \{0, 1\}$
- Saturation experiments: allow for arbitrary probabilities $\pi_k \in (0, 1)$ (Baird et al., 2018)

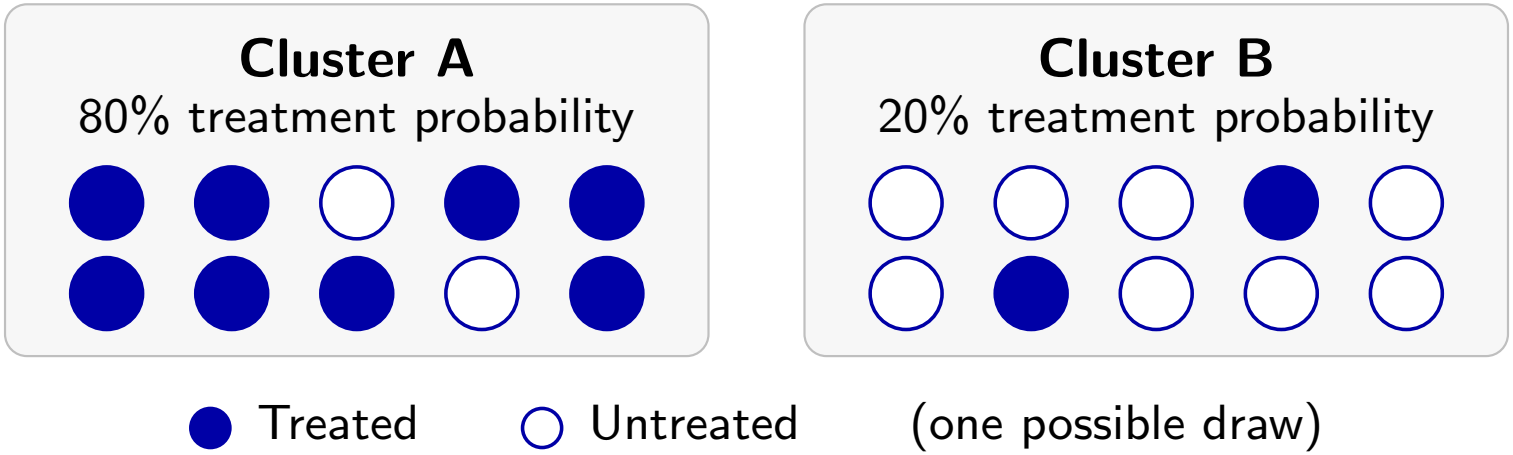
Saturation experiment: 80% vs. 20%

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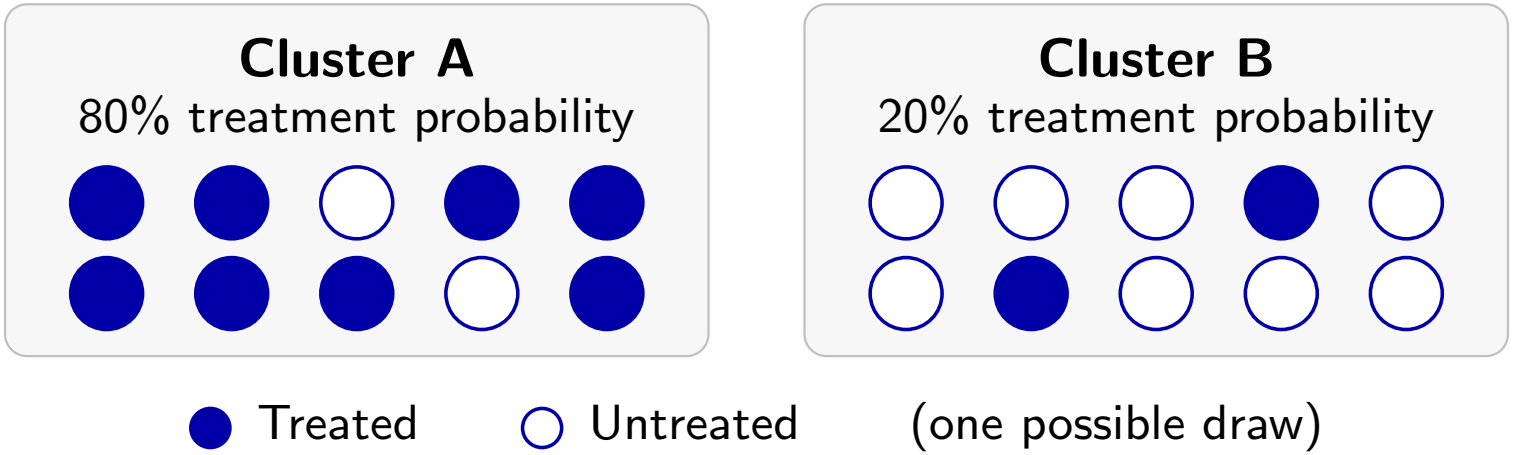
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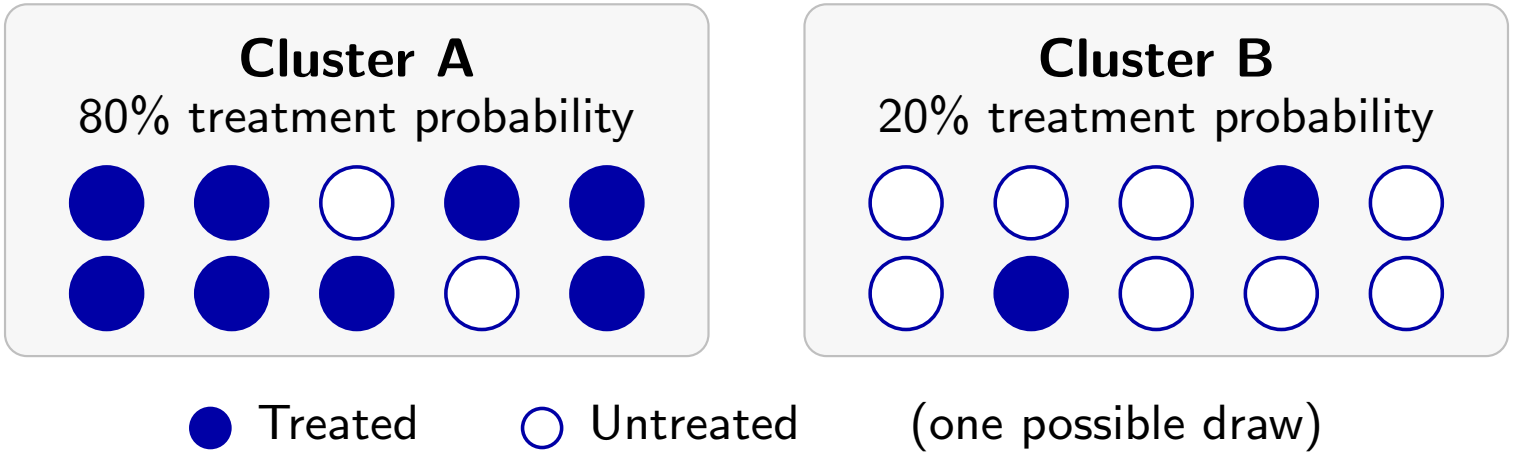
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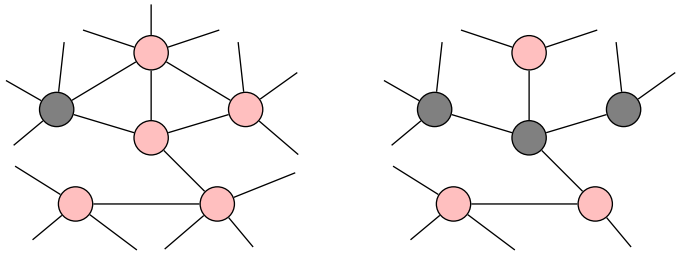
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- **Overall policy contrast:**

$$\tau_{\text{overall}}(\pi, \pi') = \mathbb{E}_\pi[Y(D_i, \mathbf{D}_{-i})] - \mathbb{E}_{\pi'}[Y(D_i, \mathbf{D}_{-i})], \qquad \pi \neq \pi'$$

Marginal effect: derivative of $\mathbb{E}_\pi[Y(D_i, \mathbf{D}_{-i})]$ with respect to π (Hu et al., 2022)

Local perturbations with clusters

- Network is difficult to collect:
  - $\Rightarrow$ Consider few unobserved networks/clusters (e.g., regions).



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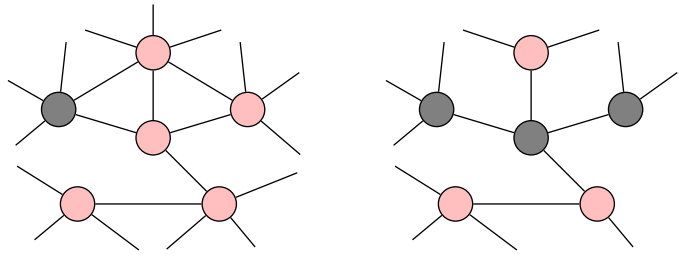
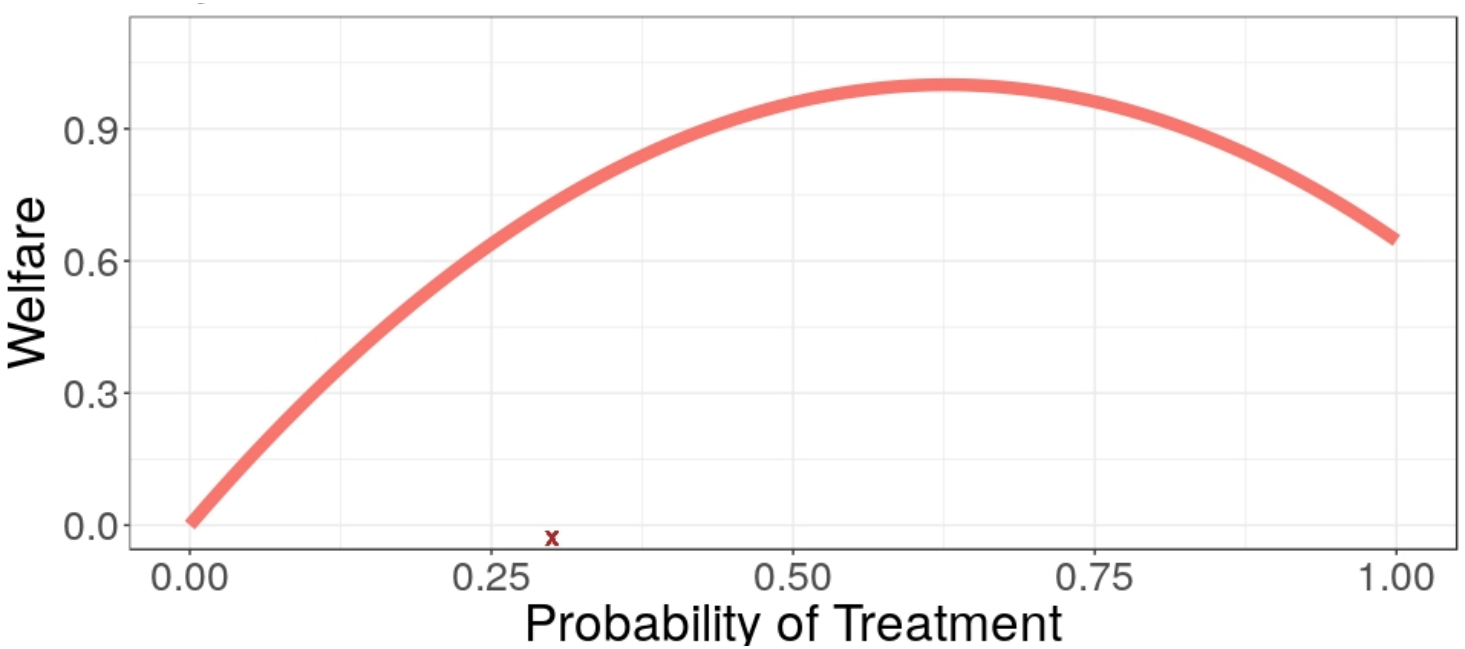


Illustration: $D_i \sim \text{Bern}(\beta)$



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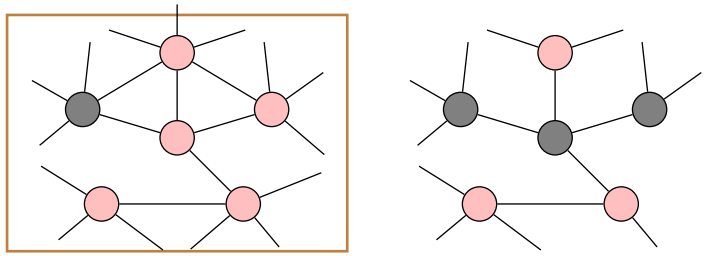
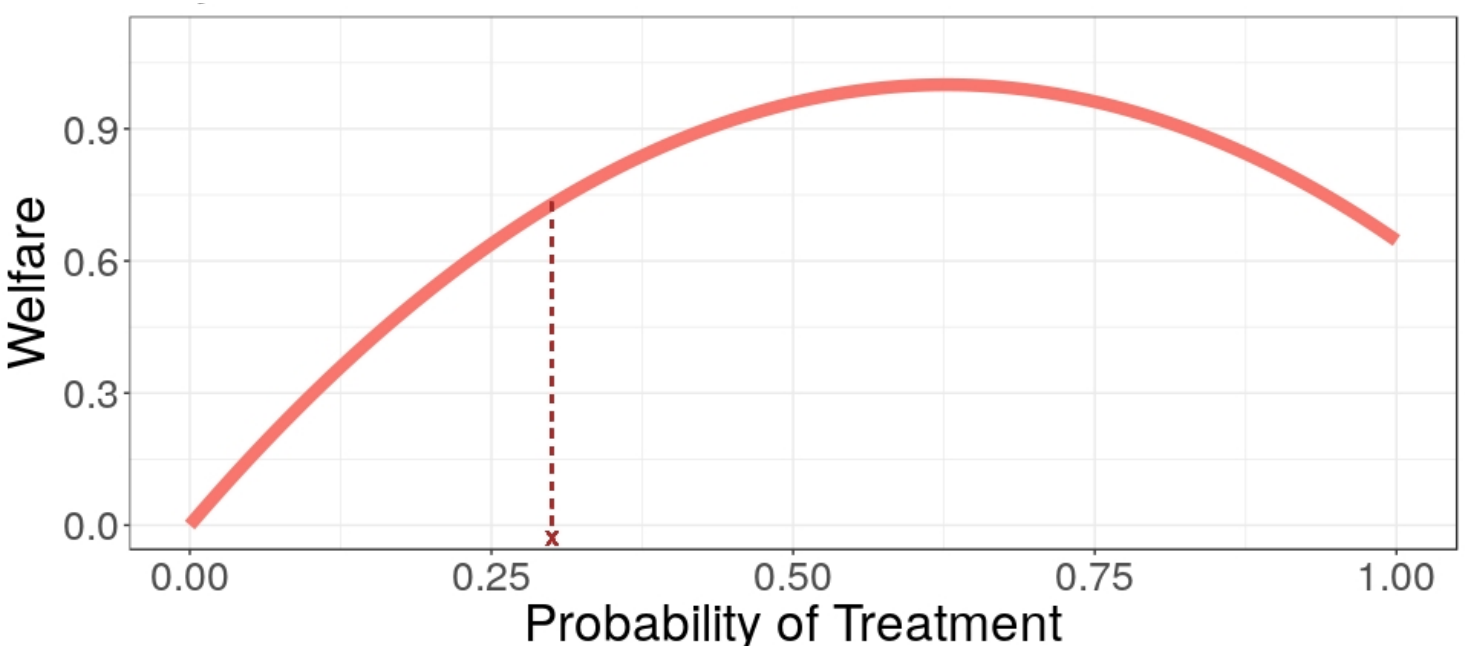


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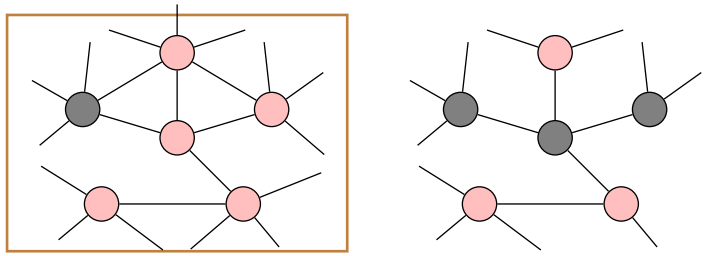
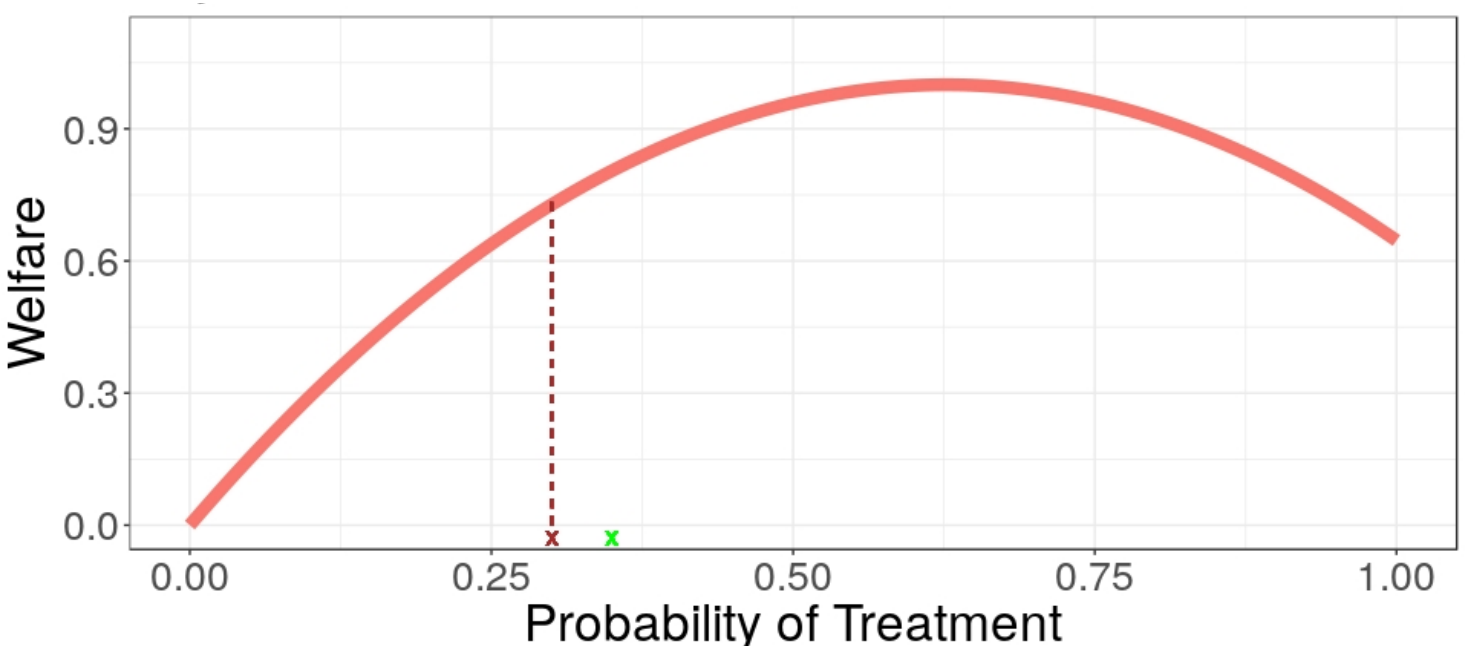


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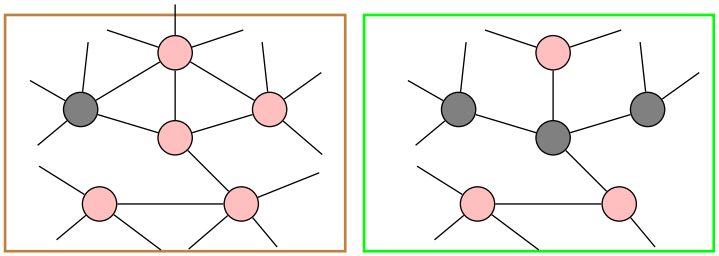
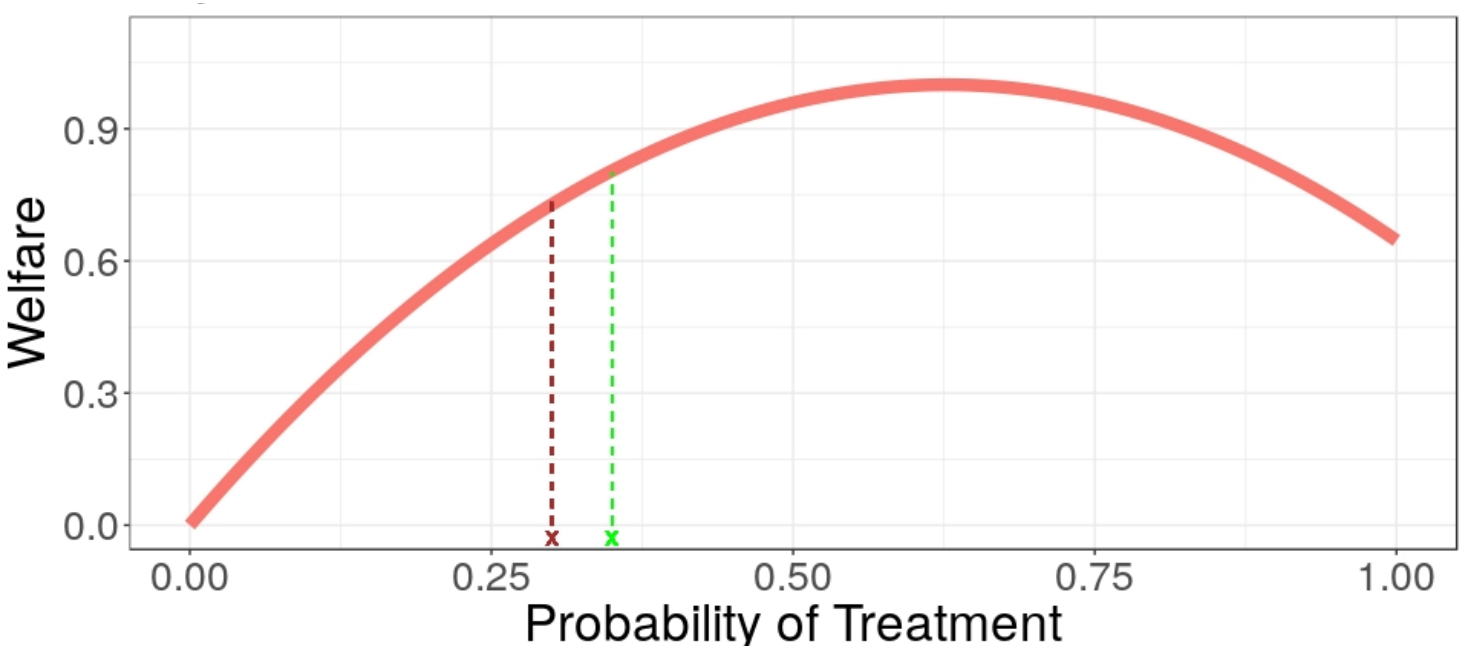


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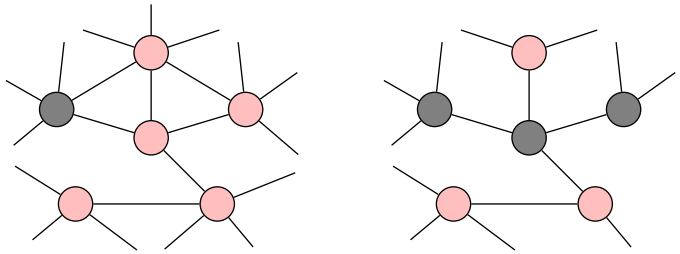
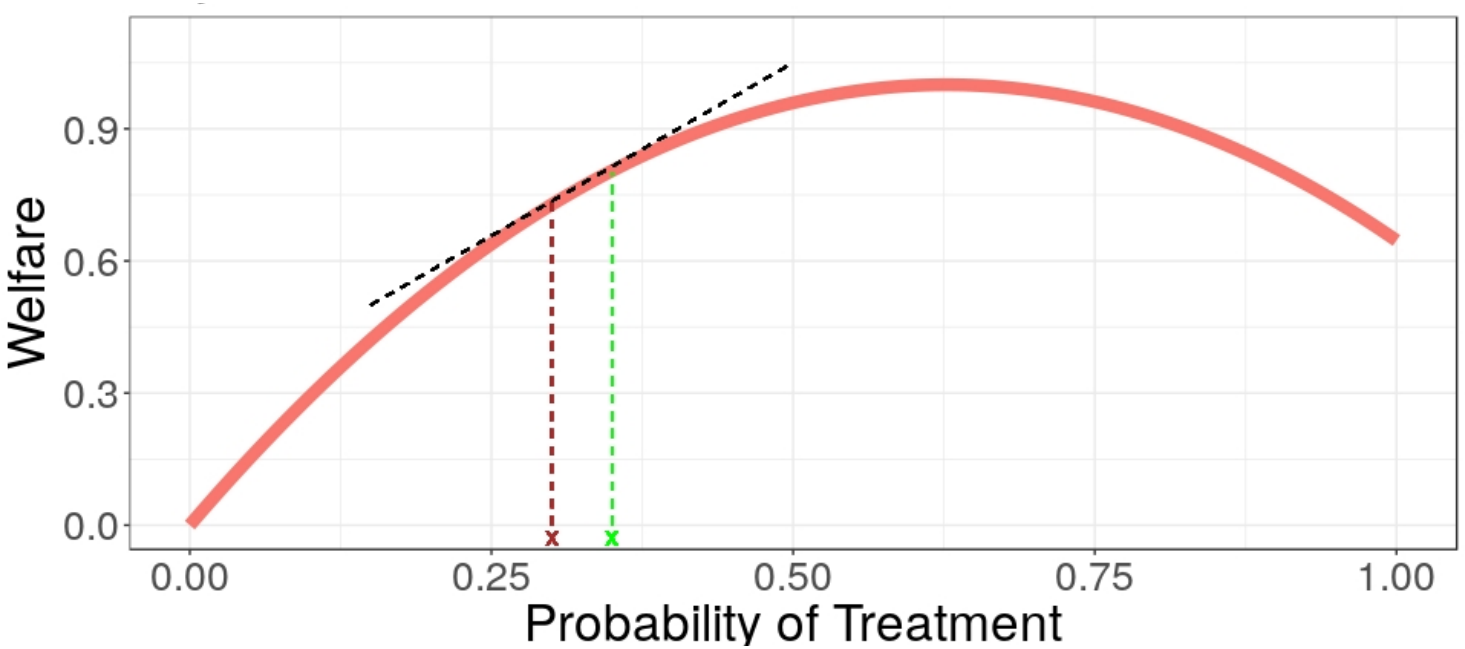


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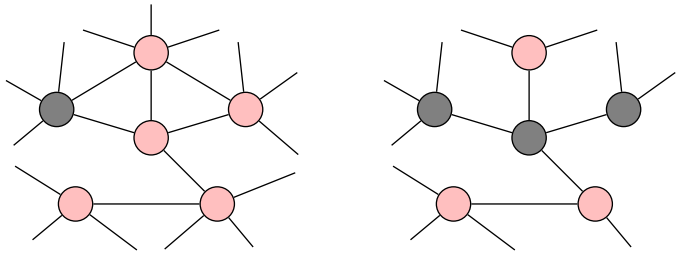
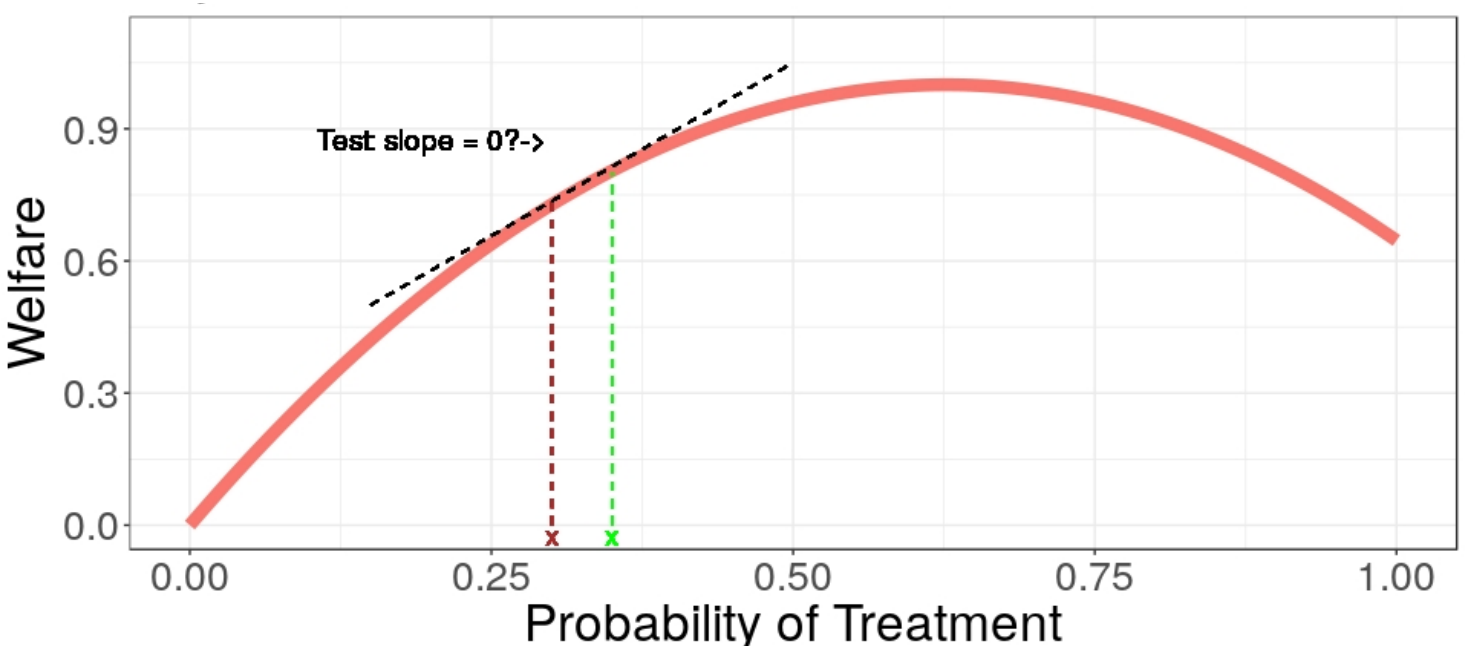


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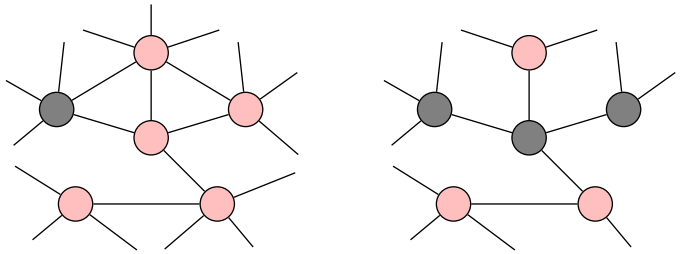
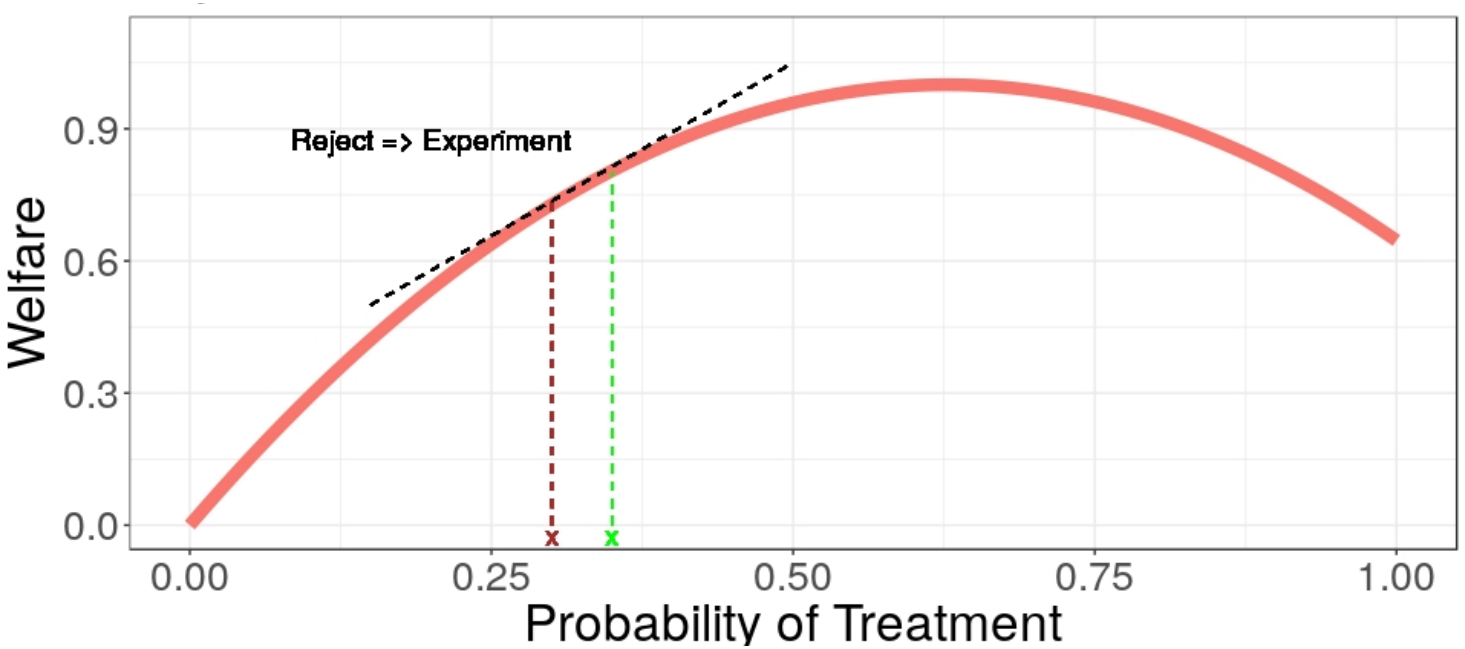


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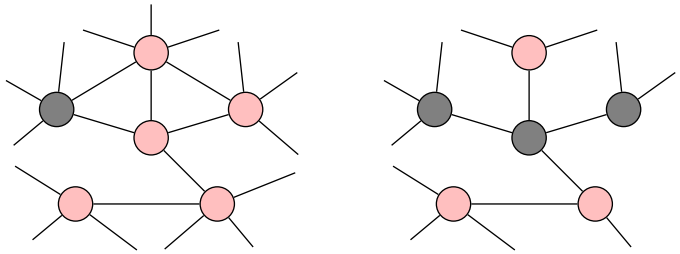
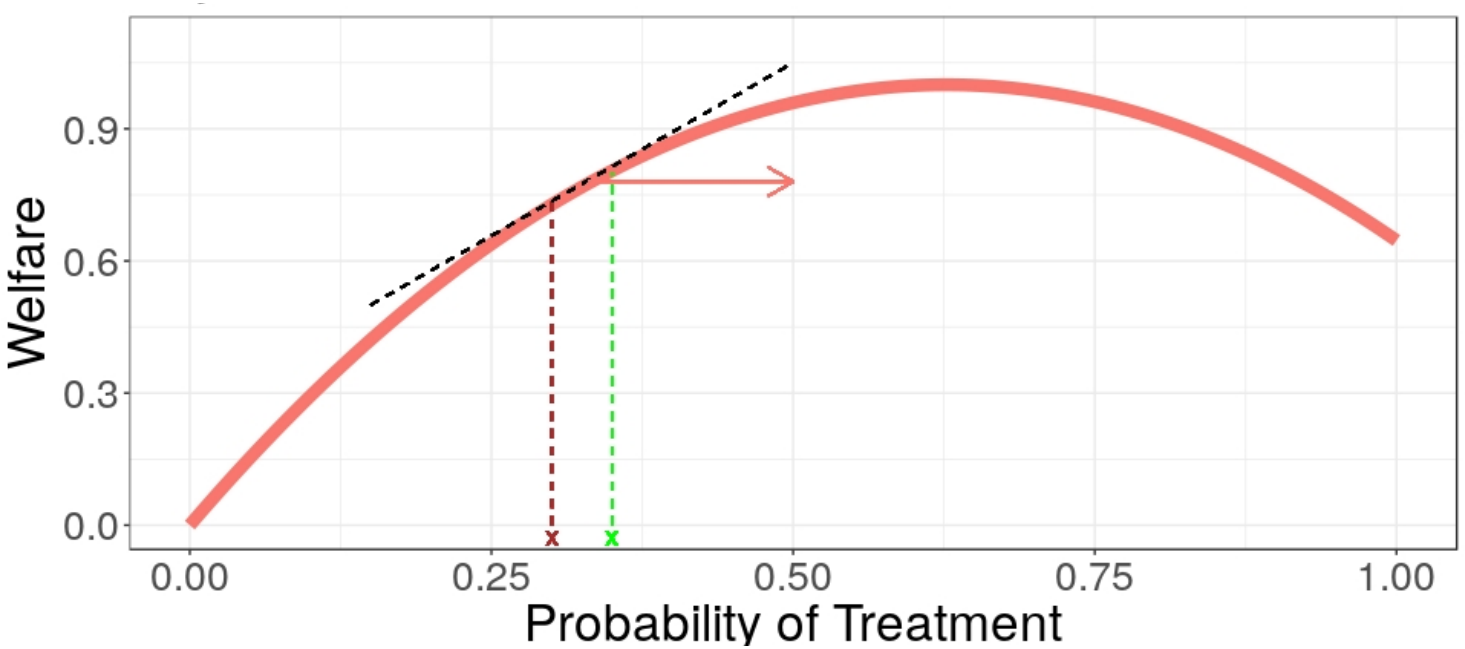
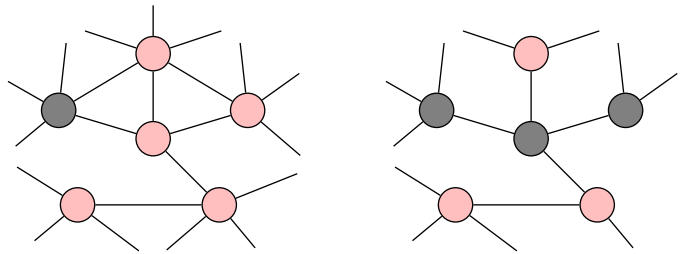


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Estimation of marginal effects

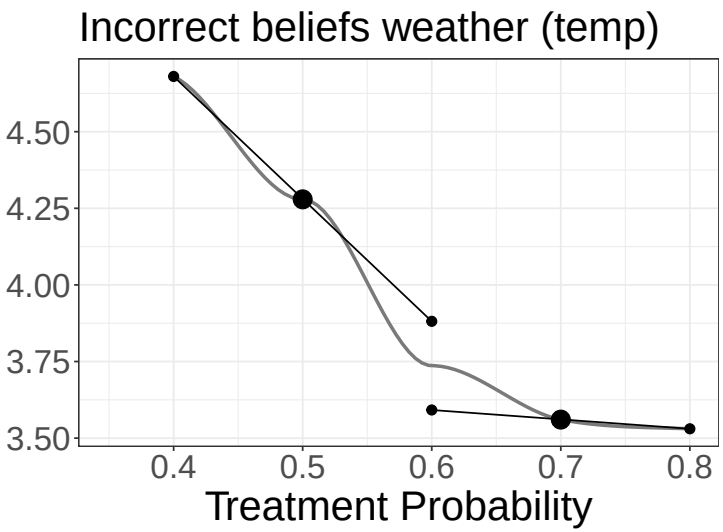
- **Estimator:** compare mean outcomes in perturbed and control clusters, divided by the change in treatment probability.
- **Consistency:** bounded-degree dependence within each cluster.
- **Inference:** approximate randomization tests with a fixed number of large clusters (Canay et al., 2017).
- **Key assumption:** homogeneous marginal effects across clusters.

An empirical example

- Study the effect of providing geo-localized weather forecasts to over 400k farmers in Pakistan (Viviano and Rudder, 2026)
- Consider different saturation levels with 10% perturbations
- Measure effects on next-day correct beliefs of farmers about weather

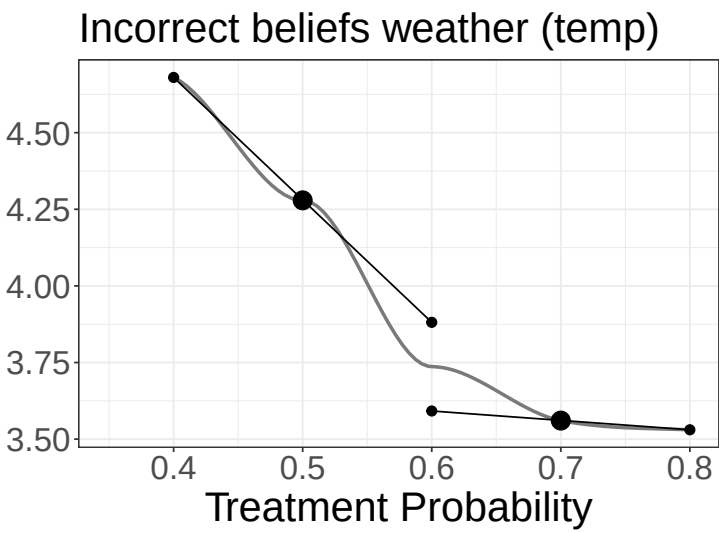
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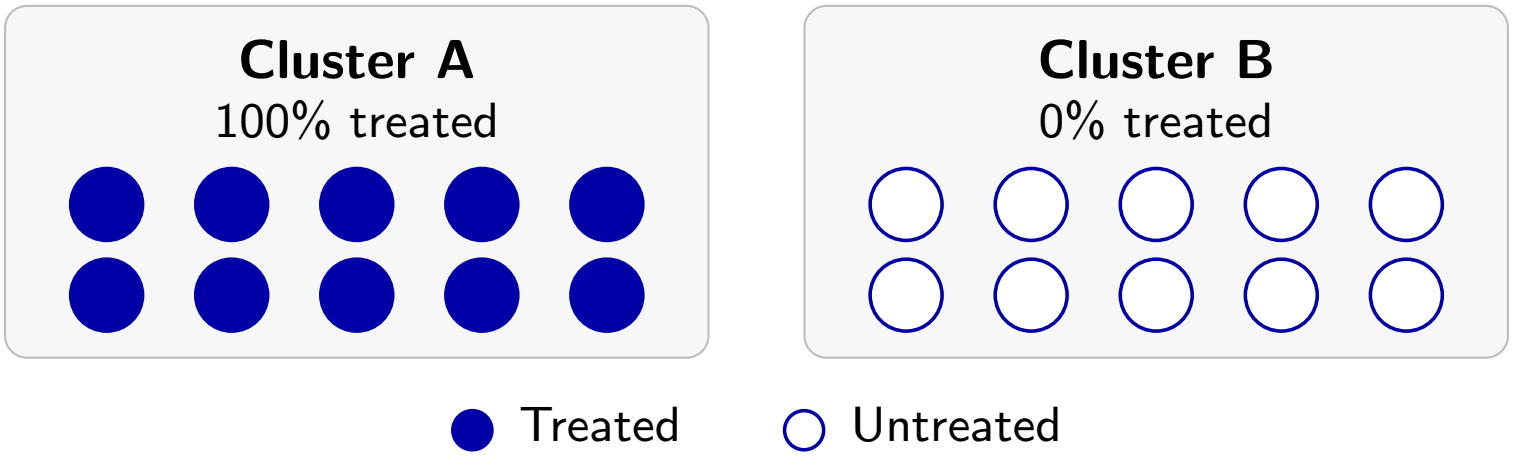
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Incorrect Beliefs about	PxD Forecast Temperature	
	$\beta = 50\%$ (Wave 1)	$\beta = 70\%$ (Wave 2)
Marginal Effect	-4.00**	-0.31
One-sided p-value	[0.02]	[0.41]
Direct Effect	-0.90*	-0.66*
One-sided p-value	[0.08]	[0.09]
Marginal Spillovers on Treated	0.43	-0.86
One-sided p-value	[0.45]	[0.37]
Marginal Spillovers on Controls	-6.39*	0.65
One-sided p-value	[0.06]	[0.43]
Observations	259	583
Note:		*p<0.1; **p<0.05; ***p<0.01

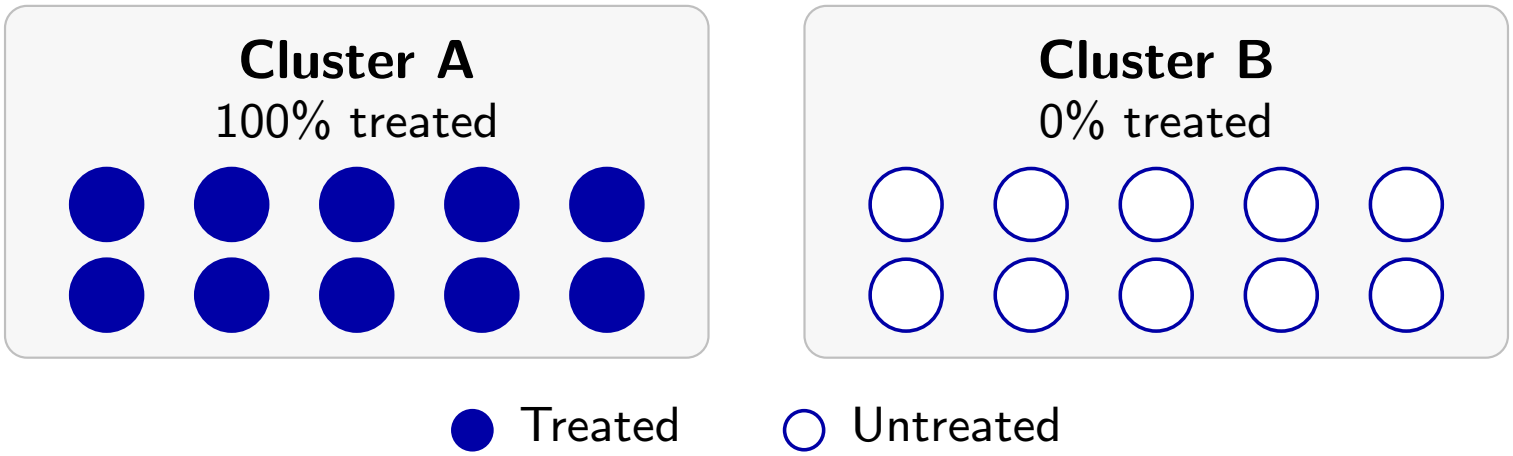
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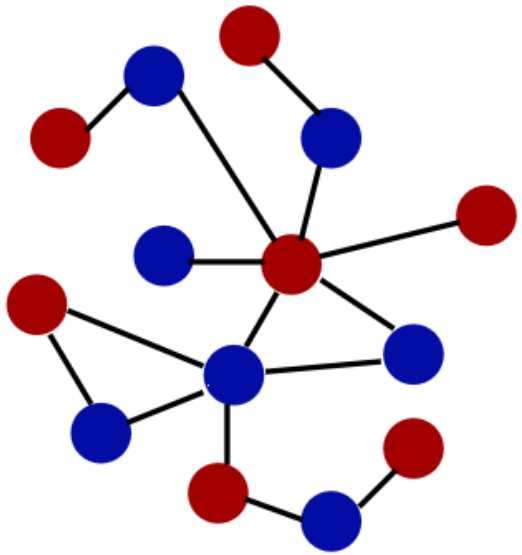


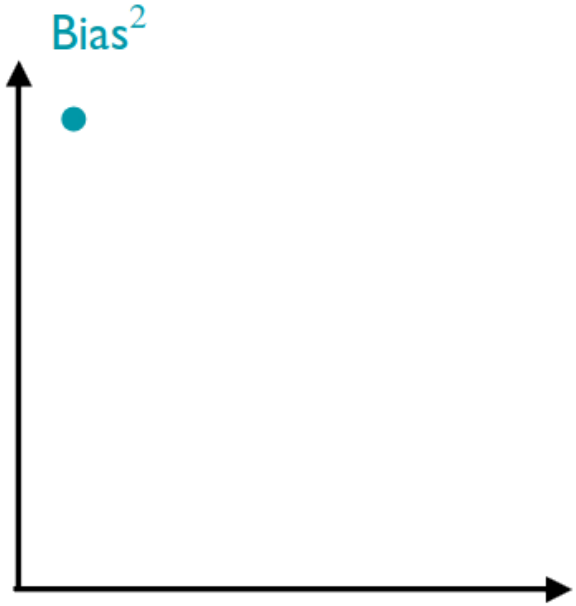
- **Typical estimator:** Difference in mean cluster outcomes across arms. Regression can adjust for baseline covariates.
- **Identifies:** The total effect of treating everyone vs. no one, assuming no spillovers across clusters

$$\tau_{\text{overall}} = \mathbb{E}[Y_i(\mathbf{1})] - \mathbb{E}[Y_i(\mathbf{0})]$$

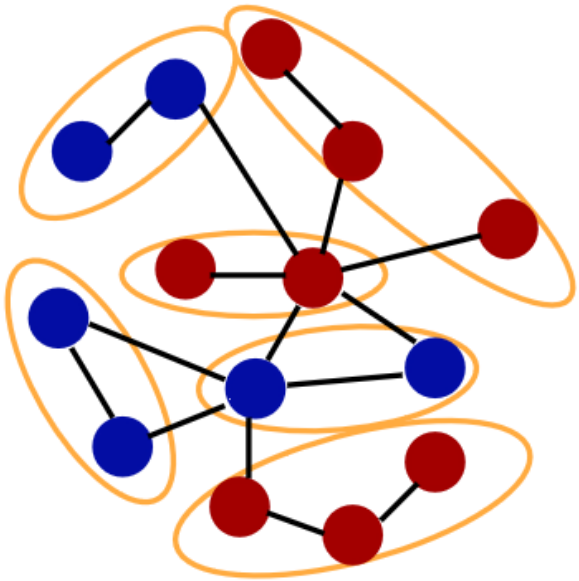
- **Problem:** Clusters may often be interconnected (with a single network)

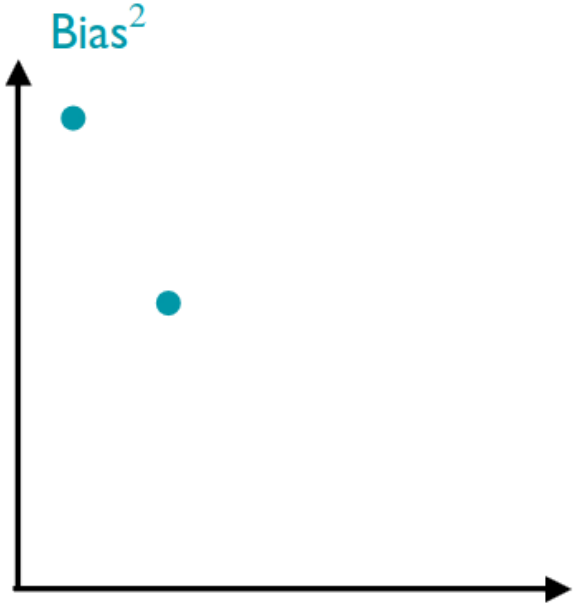
Bias-variance tradeoff for overall treatment effect



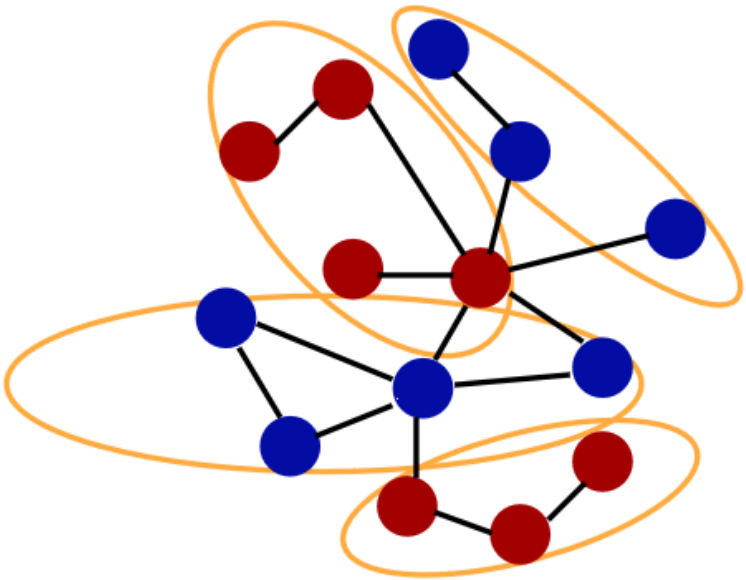


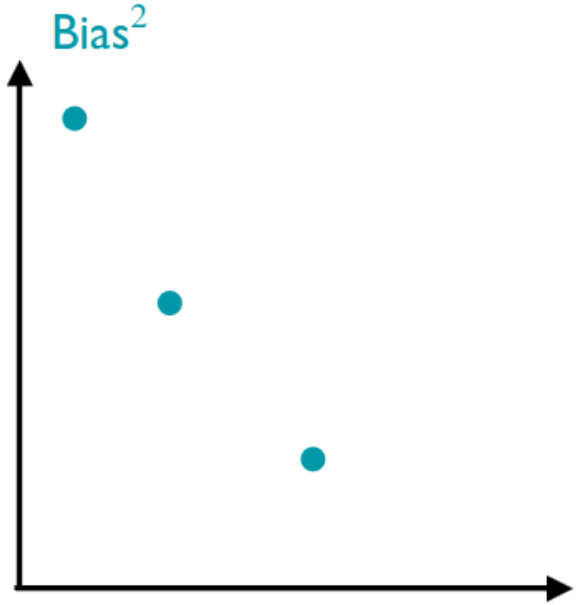
Bias-variance tradeoff with network spillovers



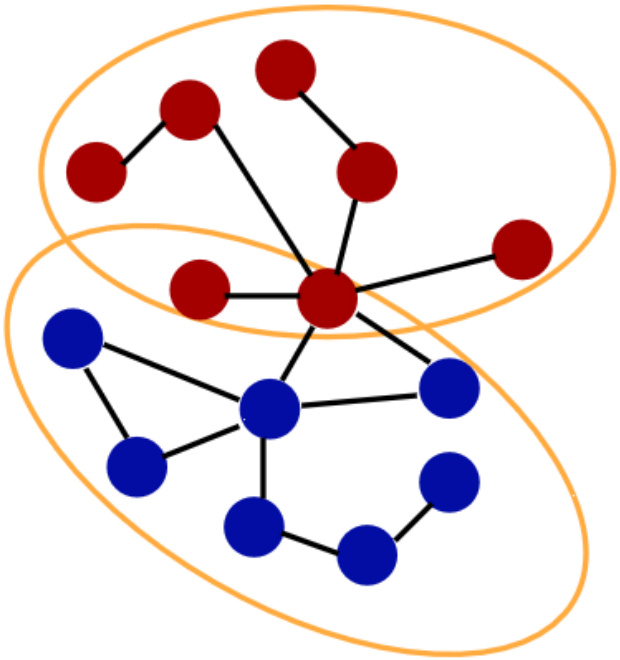


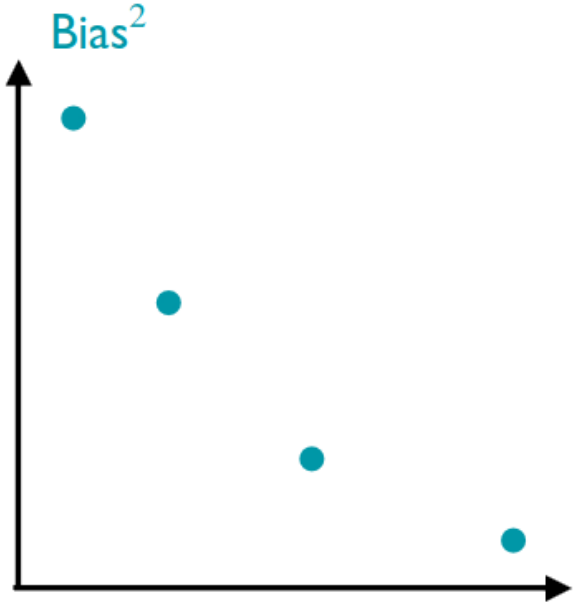
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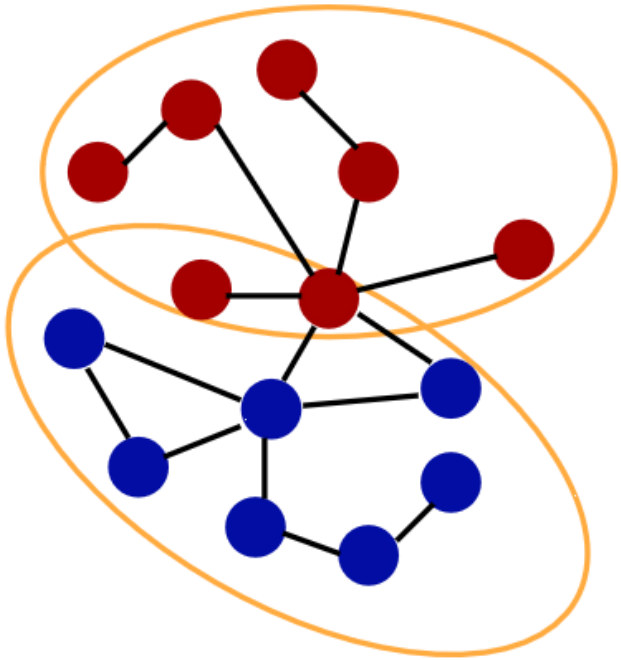


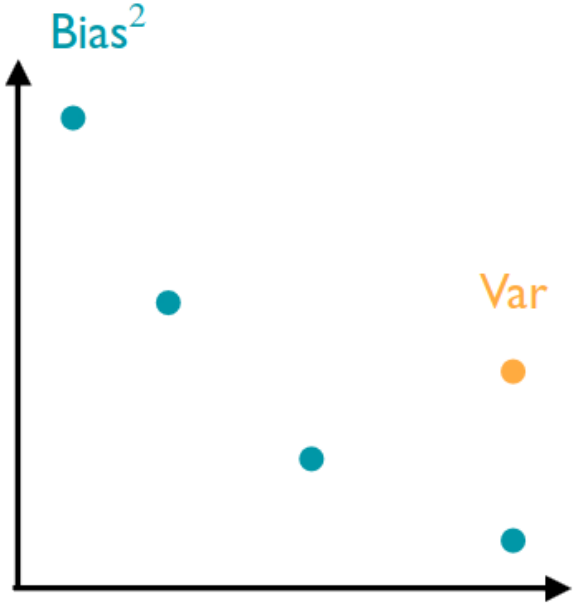
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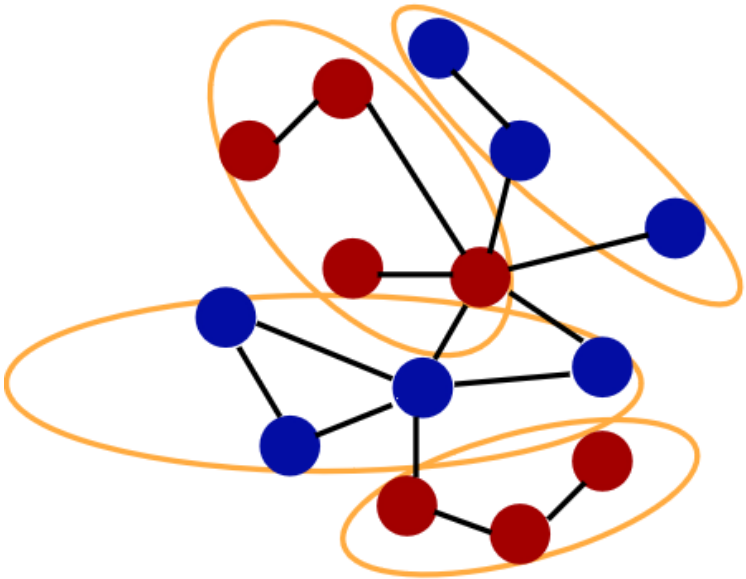


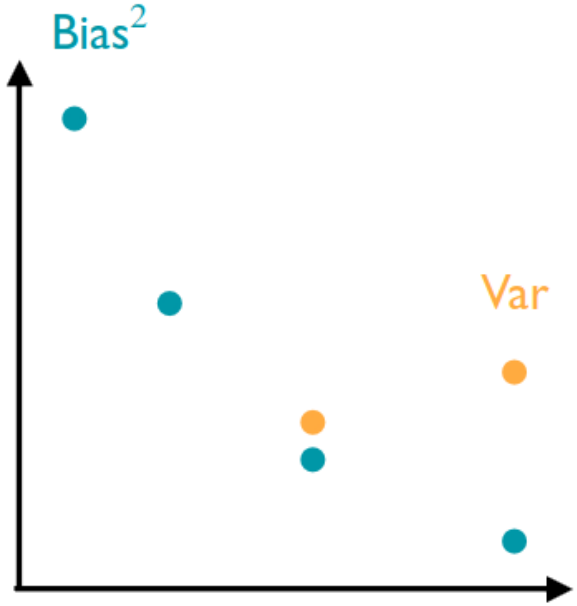
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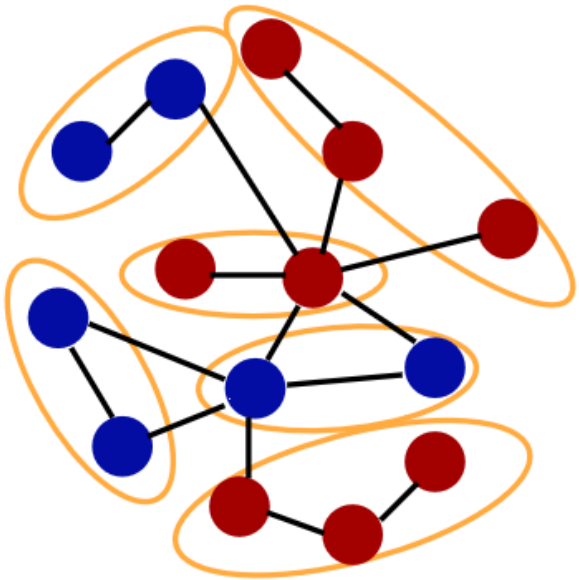


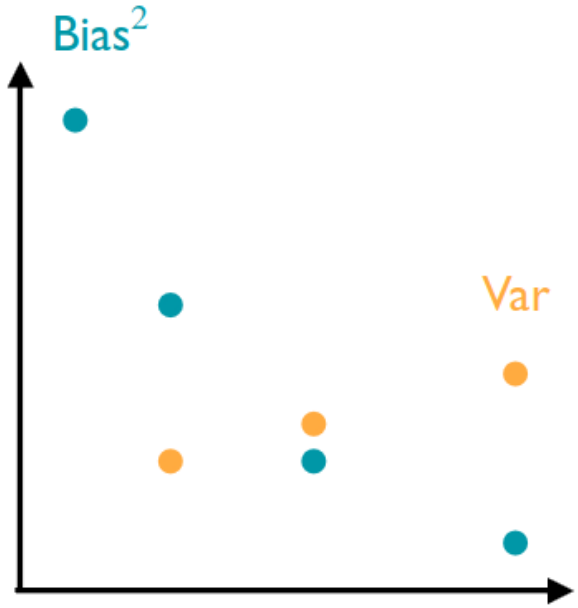
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Bias-variance tradeoff with network spillovers





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- One objective is to minimize worst-case MSE

$$\overline{\text{MSE}} := \sup_{Y(\cdot)\in\mathcal{M}^n} \mathbb{E}\left[\left(\hat{\tau} - \tau_{\text{overall}} \right)^2 \right]$$

Bias/variance characterization

- We can write (Viviano, Lei, Imbens, Karrer, Schrijvers and Shi, 2023)

$$\overline{MSE} \propto |b|^2 + \xi v^2 + \mathcal{O}(\varepsilon), \qquad \varepsilon = \frac{\bar{\phi} \cdot \text{ max degree}^2}{\#\text{clusters}}$$

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- v^2 of order $1/K$ with K number of clusters when cluster sizes are proportional
- Small (but non-negligle) $\bar{\phi}$ + sparse network $\Rightarrow$ small ε
- Idea: solve for effects $\bar{\phi}$ are local to zero (but non-negligible for inference)

Clustering vs Bernoulli design

- A common question I saw in practice

"Should I run a bernoulli or a cluster design?"

$\Rightarrow$ A Bernoulli design will ignore the bias and minimize the variance
$\Rightarrow$ Consider a cluster design with given clustering and K clusters

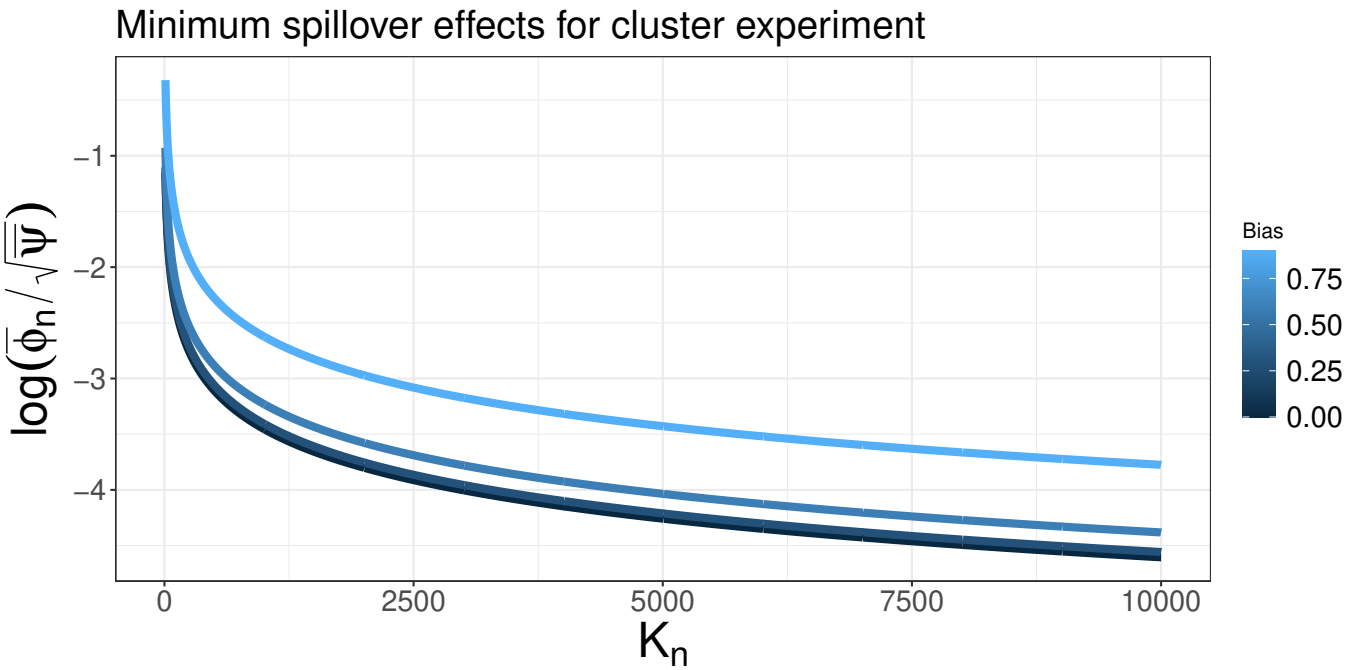
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$\Rightarrow$ On FB graph run a cluster experiment if $\bar{\phi}_n \geq 0.3\%$ for binary outcomes

Q What if I want to choose my own design of clusters?

Semi-definite optimization program

$$\mathbf{M}_{c,K} = \begin{bmatrix} 1\{c(1)=1\} & \cdots & 1\{c(1)=K\} \\ \vdots & \ldots & \vdots \\ 1\{c(n)=1\} & \cdots & 1\{c(n)=K\} \end{bmatrix}, \qquad \mathbf{L}_{i,j} = \frac{1\{i \text{ is friend with j}\}}{\#\text{ friends of } i}$$

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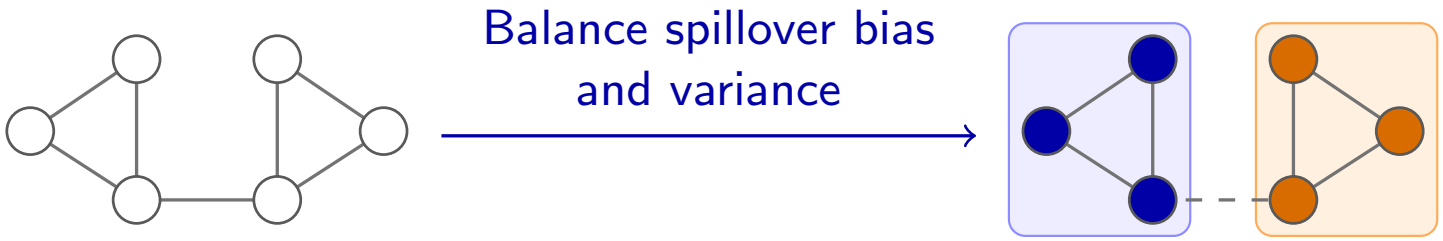
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- Formal bound on optimization error in the paper [R package online]

Implementation in practice (Semi-definite program)

Inputs: `W`, a symmetric 0/1 adjacency matrix with one row and column per unit, and a spillover weight: `calibration_grid` $= 1/\xi$ (plausible ranges).



Network: `W` Partition: `fit$clusters`

```
fit <- CausalClustering::causal_clustering_algorithm(
    W, calibration_grid = c(0.5, 1, 2), min_k = 10, max_k = 100, seed =
1)
```

- **Options:** Set feasible bounds `min_k` and `max_k`.
- **Before randomizing:** Refit at plausible calibration parameters. The package returns the cluster size and composition that you can plot.

R Package link

Empirical illustration using data from a field experiment

- Cai et al. (2015): info experiment to increase insurance take-up among farmers (48 disconnected regions, 184 connected villages)
- Two exercises:
  - Empirical evaluation of the objective function averaged over regions
  - Calibrated simulations for MSE comparisons

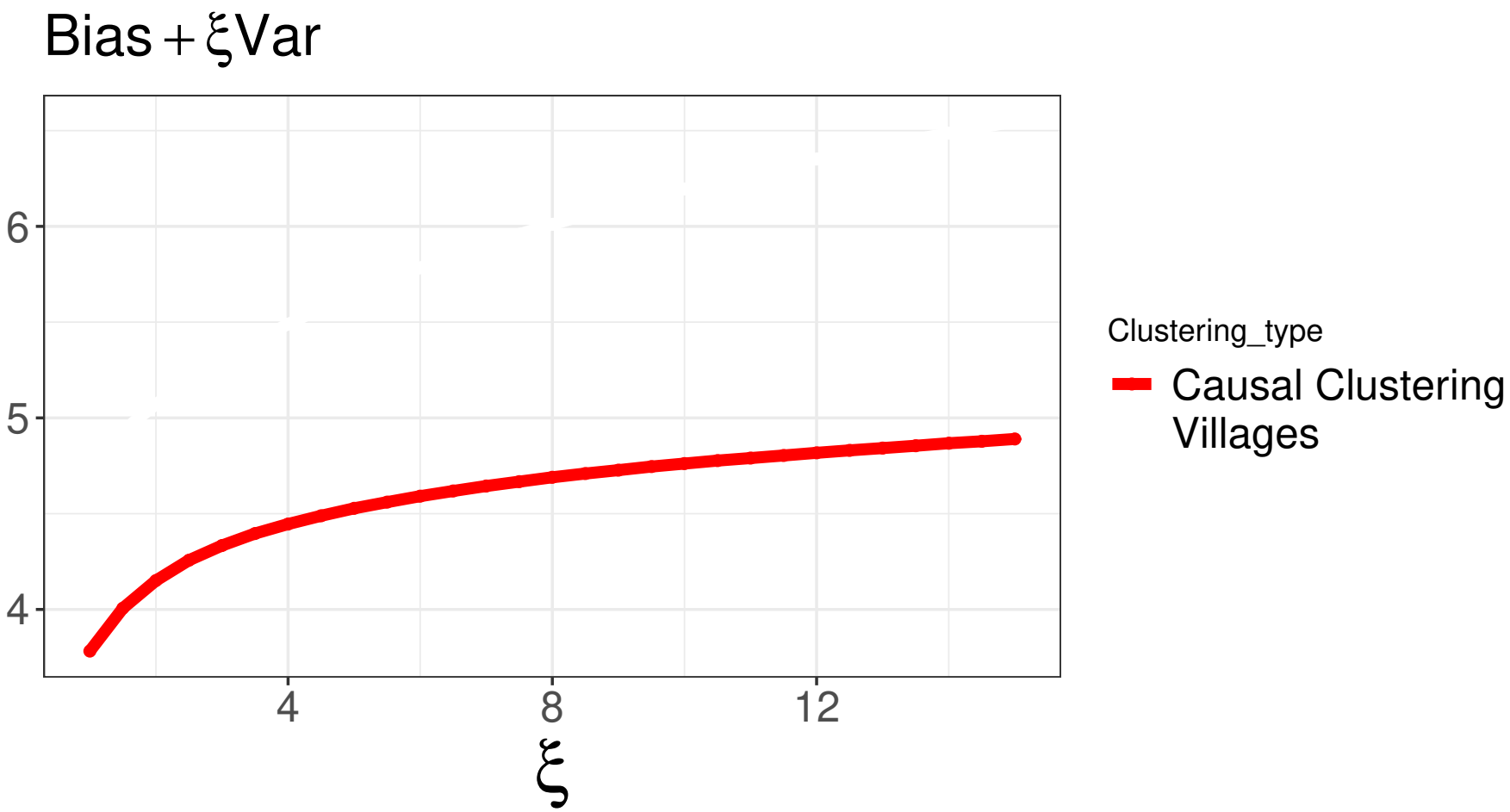
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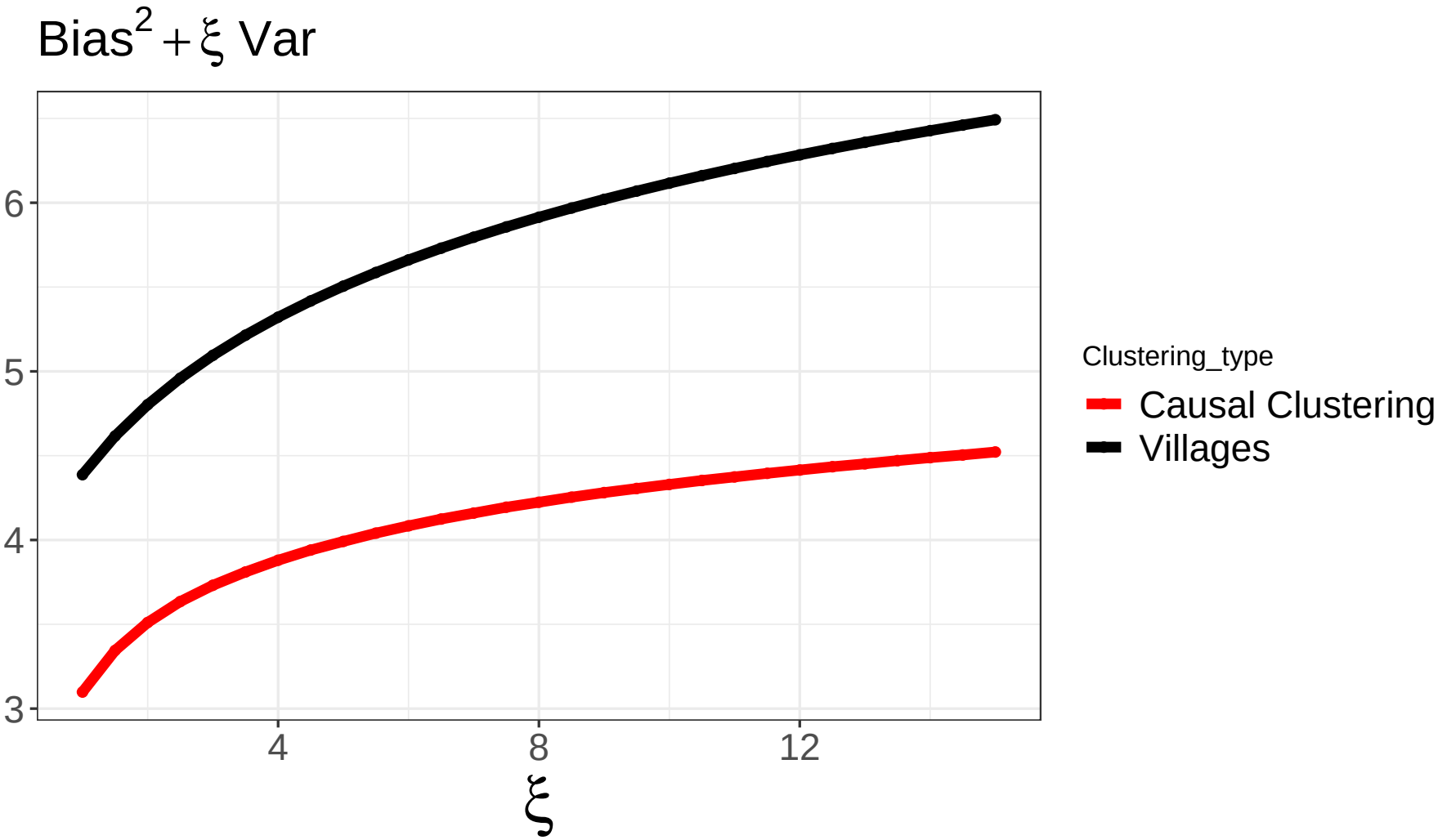
Summary

- Using villages as clusters is sub-optimal (too few villages)
- The number clusters should be 23% of observations
- Strong performance of the method especially with low/medium signal

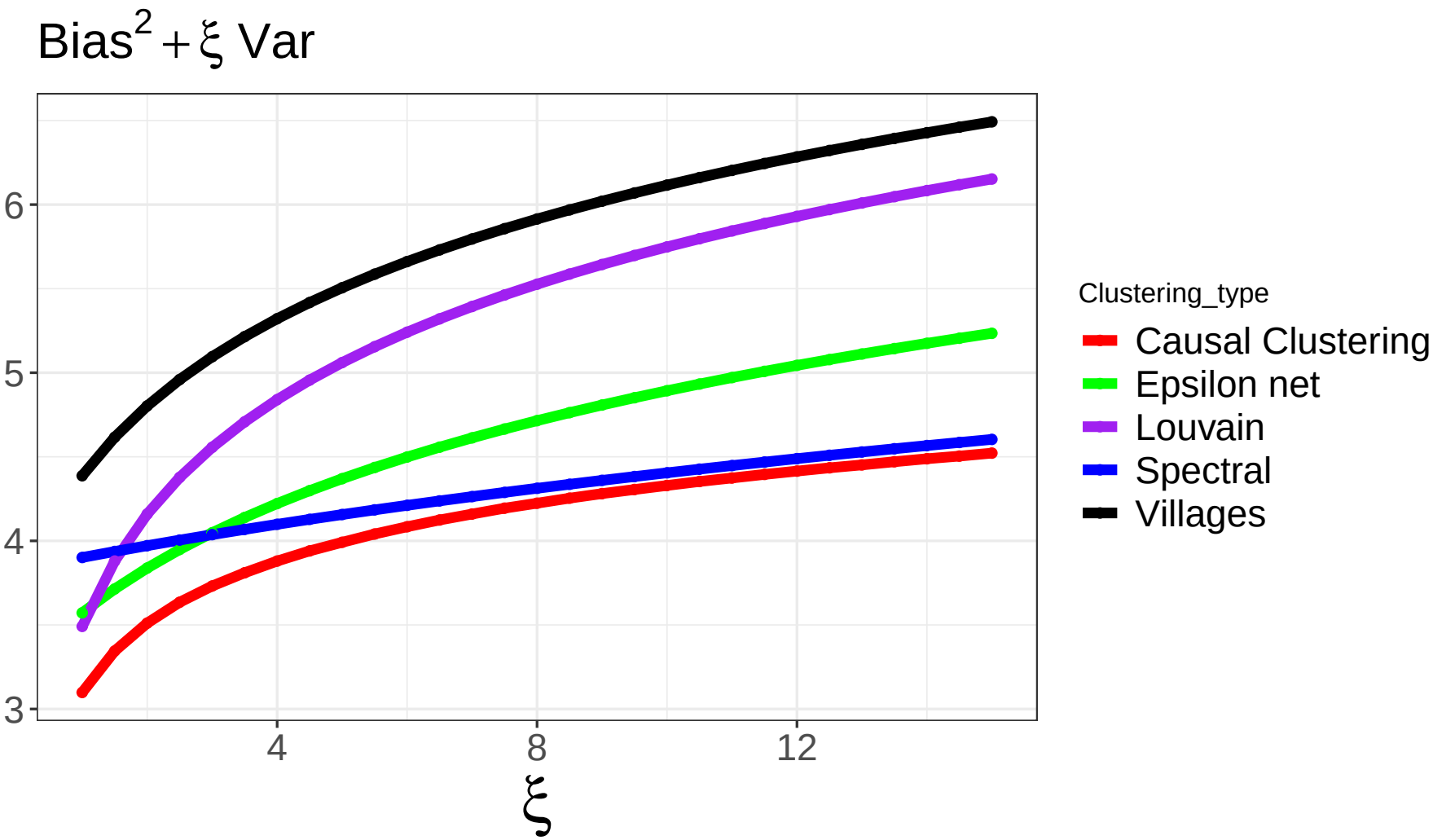
Objective function comparisons



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MSE Comparison using Calibrated Simulations

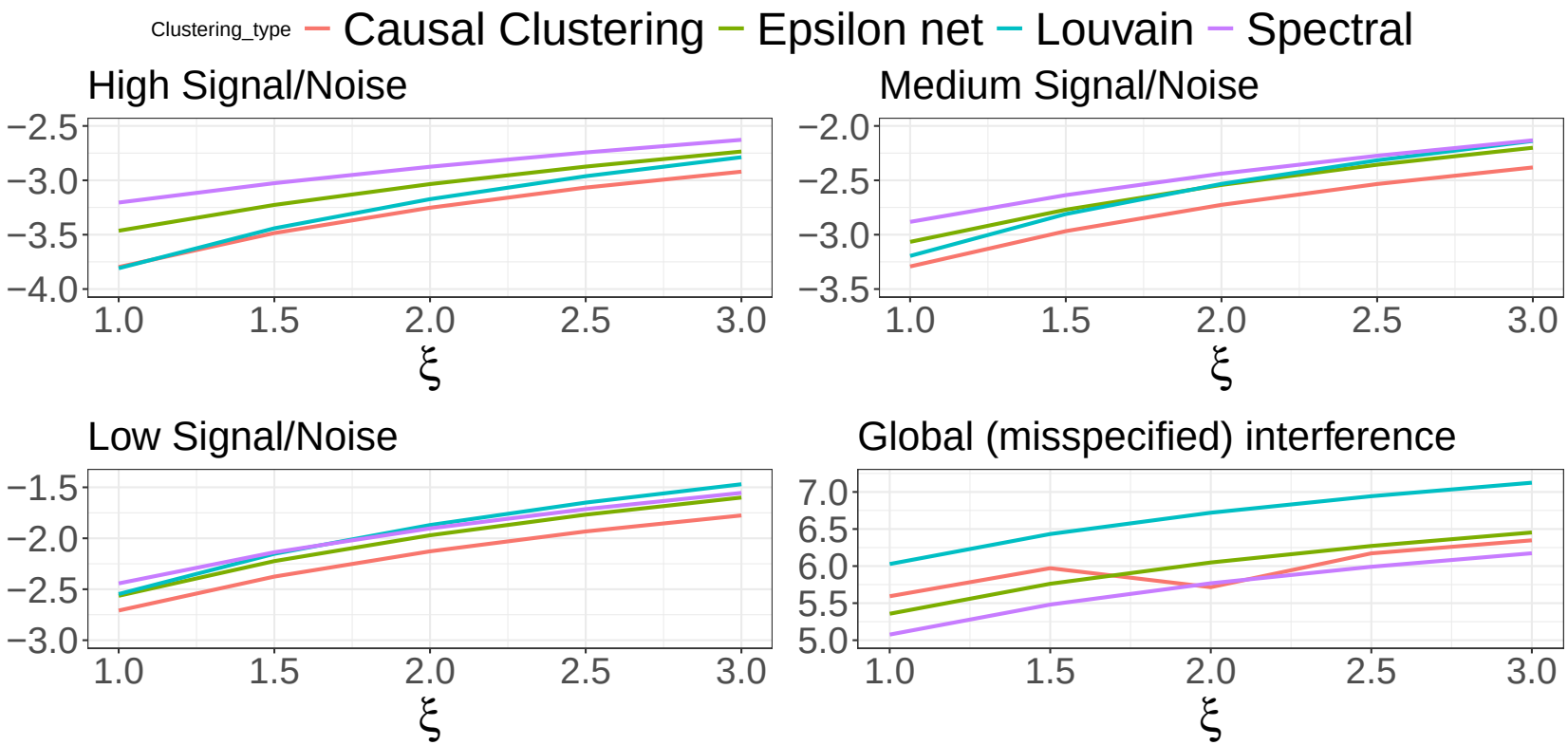
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Some related literature

- Experimental design with spillovers (Baird et al., 2018; Basse and Airoldi, 2018; Johari et al., 2022; Viviano, 2026; Pouget-Abadie et al., 2019)
- Inference under interference (Hudgens and Halloran, 2008; Aronow and Samii, 2017; Leung, 2020; Athey et al., 2018; Sävje et al., 2021; Li and Wager, 2022; Forastiere et al., 2021; Toulis and Kao, 2013)
- Inference with clusters (Abadie et al., 2023; Ibragimov and Müller, 2016; Canay et al., 2017)

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- Clustering in stat (von Luxburg, 2007; Newman, 2013; Lei et al., 2020)

  $\Rightarrow$ Here different focus on clustering for causal inference

Content

1 Setup and overview of recent literature

2 Topics on network experiments

3 Policy design

Policy Targeting

Suppose we collect data from experiment or quasi-experiment participants sampled from a large population

How do we *design* an allocation rule (policy function) that determines the treatment for the entire population?

Key features:

- Heterogeneity in treatment effects
- Network interference
- Treatment may be costly

Constraints may apply:

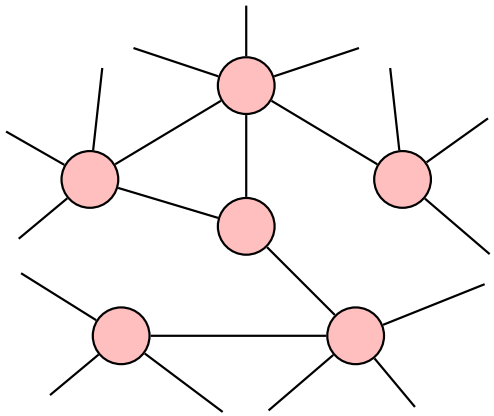
- Network information may not be observable in other villages
- Ethical, legal or computational constraints may apply

Policy Choice: Illustration



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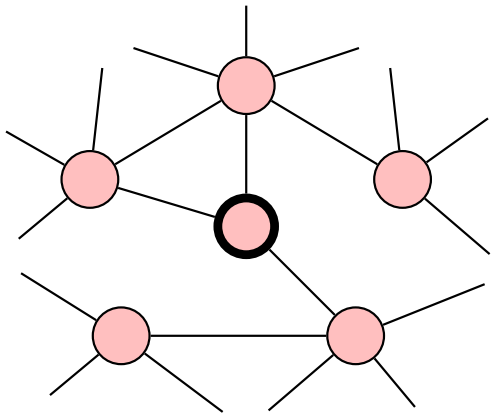




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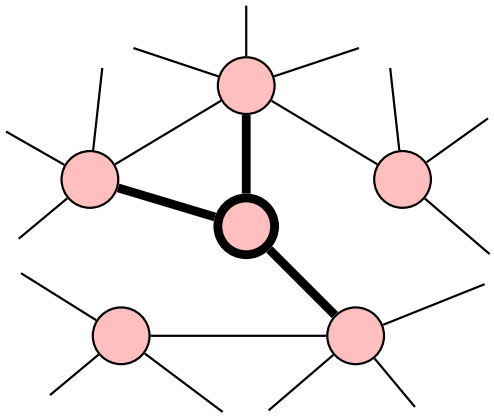
Randomly select participants



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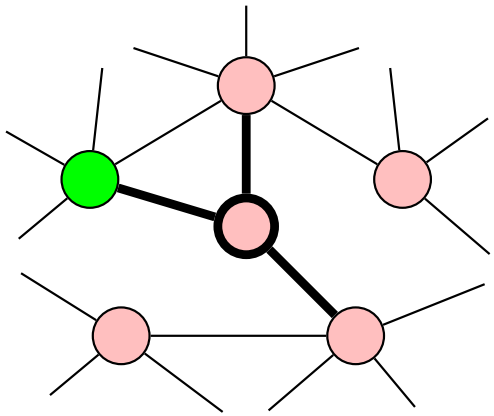
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Policy Choice: Illustration



Randomize treatment
for participants and neighbors

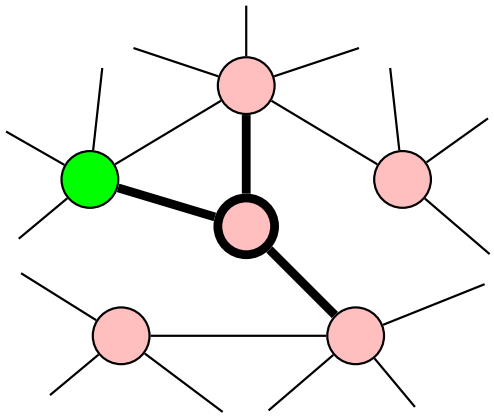


Observe participants' outcomes
and both groups' covariates
and treatment assignments

Policy Choice: Illustration

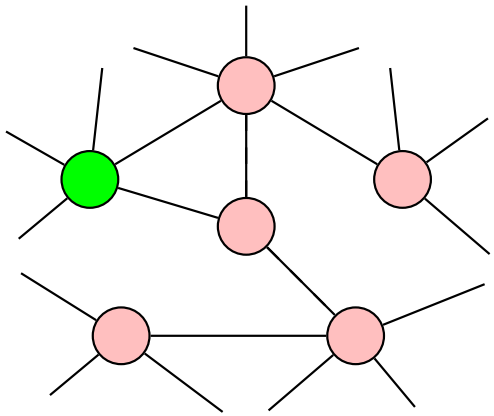


Randomize treatment for participants and neighbors



Observe participants' outcomes and both groups' covariates and treatment assignments

Choose a targeting rule

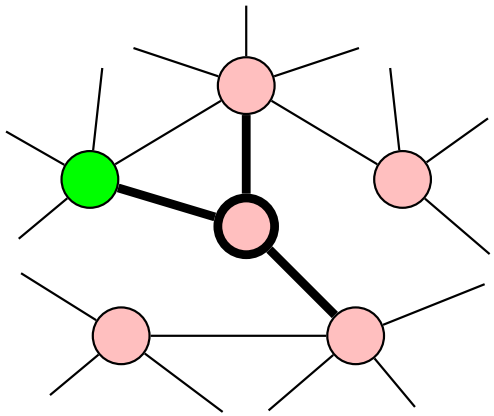


Covariates observed for the whole population

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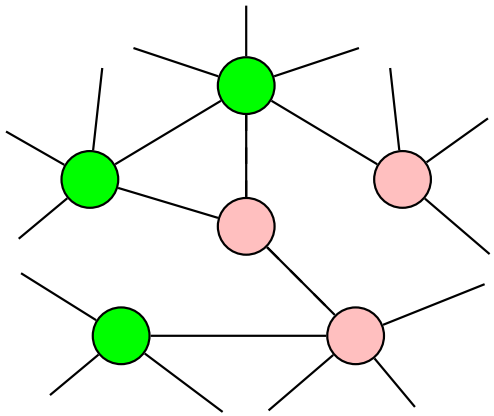


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Covariates observed for the whole population

Illustrative Example

- Hard problem, often leverage explicit centrality measures (Bloch et al., 2023)

$\Rightarrow$ Reduce interference to local and leverage ML models to target on observables

- Consider a policy $\pi(X_i) \in \{0, 1\}$ choosing whether to treat each individual with baseline observables X_i (e.g., covariates, neighbors' covariates, centrality measure,...)
- Experiment randomize treatments and model

$$Y_i(d_i, \mathbf{d}_{-i}): \text{ function of ego treatment, friends' treatments, covariates, ...}$$

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$$Y_i(d_i, \mathbf{d}_{-i}): \text{ function of ego treatment, friends' treatments, covariates, ...}$$

- Our goal is to maximize the average outcome in population Viviano (2025)

$$W(\pi) = \frac{1}{n} \sum_i \mathbb{E}_Y \Big[Y_i \Big(\underbrace{\pi(X_i)}_{\text{Ego treat under } \pi}, \underbrace{\boldsymbol{\pi}(X_{-i})}_{\text{Friends' treatment under policy } \pi} \Big) \Big]$$

$\Rightarrow$ Estimate an unbiased target using main participants and optimize over π

$\Rightarrow$ Allow for constraints on $\pi \in \Pi$

Sampling and experiment

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$\Rightarrow$ Example

- Randomly select a subset of individuals from population
- Randomize treatments to such individuals and their friends
- Treatments for the remaining units equal to baseline $D_i = 0$.

Outcomes

$$Y_i = r\Big(D_i, T_i, X_i, |\mathcal{N}_i|, \varepsilon_i\Big), \quad T_i = g_n\Big(\sum_{k \in \mathcal{N}_i} D_k, X_i, |\mathcal{N}_i|\Big)$$

for $T_i \in \mathcal{T}_n$, $r(\cdot)$ unknown and $g_n(\cdot)$ known

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(C) $\varepsilon_i | A, X$ dependent for one/two-degree neighbors only

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- Comments
  - The policy can use network features in X_i when network data are available at rollout
  - Welfare defn implies no carry-overs from previous experiment
  - Policies $\pi(D_i, X_i) = D_i + (1 - D_i)\pi(X_i)$ are possible

Estimands

Defn Conditional Mean

$$m(d, t, x, l) = \mathbb{E}\Big[r(d, t, X_i, |N_i|, \varepsilon_i)\Big| X_i = x, |N_i| = l\Big]$$

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$\Rightarrow$ No assumptions on (A, X, ε_i)

Estimation with known propensity score

- (Augmented) Inverse probability weighting:

$$W_n(\pi, m^c, e) = \frac{1}{n_e} \sum_{i=1}^{n} R_i \left(\frac{I_i(\pi)}{e_i(\pi)} \Big(Y_i - m_i^c(\pi) \Big) + m_i^c(\pi) \right),$$

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- Optimization: estimate $\hat{\pi}_{m^c,e}$ by maximizing $W_n(\pi, m^c, e)$
  - $\Rightarrow$ show that optimization admits a mixed-integer linear program for a large class of function classes Π
  - $\Rightarrow$ R-package implement a smooth gradient descent version

- Require overlap condition $e_i \in (\delta, 1-\delta), \delta \in (0, 1)$

Regret bound: known propensity

Theorem 3.1

For Y_i having bounded third moment given

$$\mathbb{E}\left[\sup_{\pi\in\Pi} W(\pi) - W(\hat{\pi}_{m^c,e})|A,X\right] = \mathcal{O}\left(\frac{max\ degree^{3/2}}{\delta_n}\sqrt{\frac{\mathrm{VC}(\Pi)}{n_e}}\right).$$

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- No restrictions on n_e relative to n (valid for $n \gg n_e$)
- Bound is informative only under "enough" sparsity
- In the paper: network cross fitting to control dependence structure

Estimation with Generic ML Algorithms

- Rate characterization also depends on convergence rate of $\hat{m}, \hat{e}$:
  - $\Rightarrow$ Minimax regret rate achieved with parametric estimators ($n^{-1/2}$);
- Can we improve the rate of convergence for semiparametric ones?
  - $\Rightarrow$ Network dependence invalidates standard cross-fitting (ML) estimators

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- Partition sampled units into groups such that in each group two sampled units are not friends or have no shared friend
- Estimation via standard cross-fitting within each group

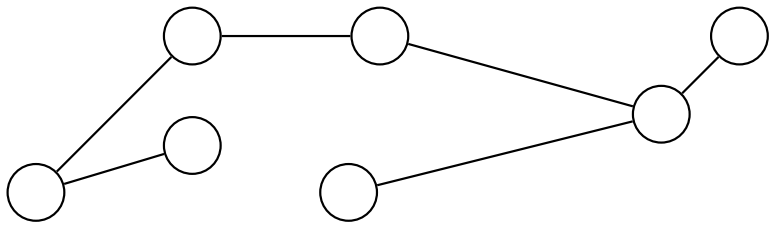
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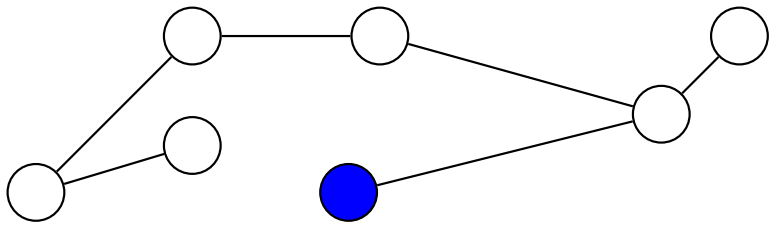
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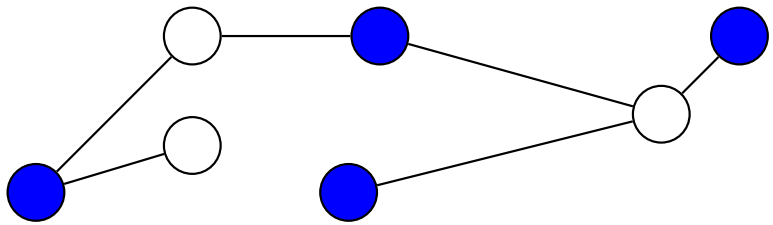
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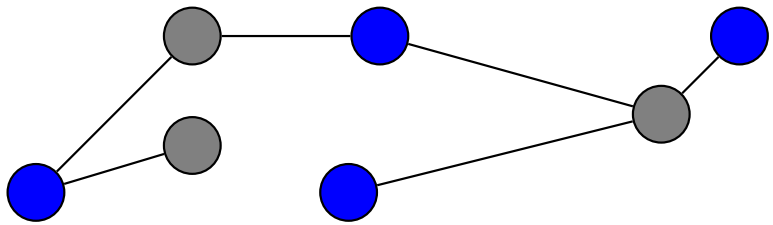
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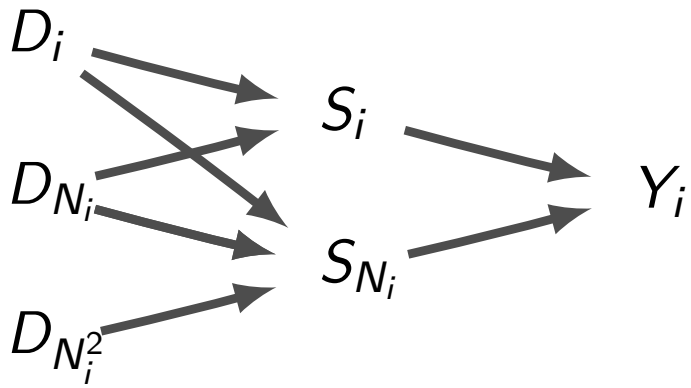
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Extensions

- Poor overlap and trimming: ignore the direct effect on largely connected nodes, but incorporate spillovers that they generate

- Non-compliance: incentive $\pi(X_i)$ generates spillovers on selection into treatment.

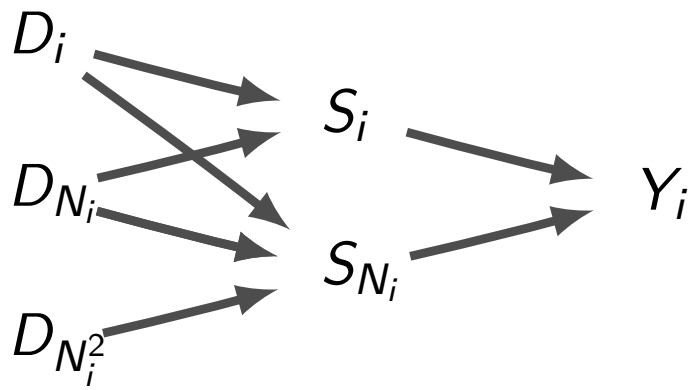


$\Rightarrow$ Simple-to-estimate expression that depends on $P(S_i = 1|\cdot)$.

Extensions

- Poor overlap and trimming: ignore the direct effect on largely connected nodes, but incorporate spillovers that they generate

- Non-compliance: incentive $\pi(X_i)$ generates spillovers on selection into treatment.



$\Rightarrow$ Simple-to-estimate expression that depends on $P(S_i = 1|\cdot)$.

- Different target population:
  - (i) reweighting mechanism for empirical welfare
  - (ii) If weights are unknown, control *expected* regret

- Higher order dependence of order M
  - Rates of convergence depend on higher order terms $\mathcal{N}_{n,\max}^{M/2}$

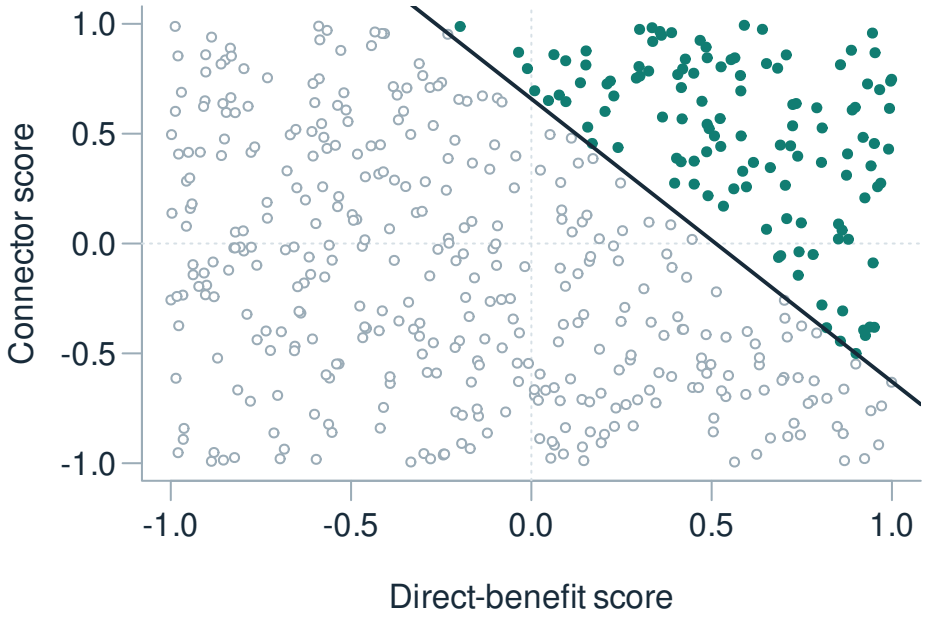
Fitting and evaluating a rule in R

```
parameters <-  list(
  model = "linear_regression", method = "surrogate",
  cost = cost, maximum_time = 120, params = tuning)
fit <- do.call(NetworkTargeting, c(package_data(train), parameters))
a <- PredictPolicies(fit, test$X)
v <- do.call(NetworkWelfare, c(package_data(test), parameters))
v$welfare
```

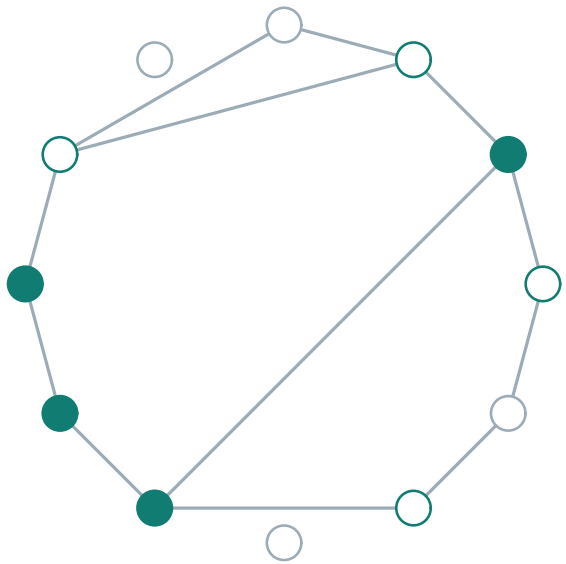
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v <- do.call(NetworkWelfare, c(package_data(test), parameters))
v$welfare
```

Eligibility on the evaluation sample



Assignments in one village



Conclusions

- Spillovers may challenge the analysis and design of experiments, but are also an opportunity to learn more (and better)
- In this presentation we reviewed some of the literature, discussed topics on experimental design and study policy targeting.

Some work in progress

- Measuring spillovers and complementarities for social norm adoptions (with Eliana la Ferrara and Cheoeon Lim)
- Identification and inference with unknown networks (with Gabriel Chodorow-Reich and Xavier Gabaix)

Thanks very much!

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